Advanced Course on Electric Drives

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Chapter 1

Introduction to Advanced Electric Drives

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Graduate Course in Electric Drives

Objectives:

- Basics to Advanced Topics in 2 Semesters
- Seamless Continuation of the First Course
- Topics: Dynamic Modeling and Control
- Approach/Tools
  - *dq*-Windings based Analysis
  - Design Examples Using Simulink
  - Verification in the Hardware Lab using dSPACE

Topics (Lectures)

1. Introduction to Advanced Electric Drive Systems (2)
2. Induction Machine Equations in Phase Quantities: Assisted by Space Vectors (5)
3. Dynamic Analysis of Ind. Mach. in terms of *dq*-Windings (7)
4. Vector Control of IM Drives: A Qual. Examination (4)
5. Mathematical Description of Vector Control (5)
6. Detuning Effects in Induction Motor Vector Control (3)
7. Space Vector PWM (SV-PWM) Inverters (3)
8. Direct Torque Control (DTC) and Encoder-Less Operation of Induction Motor Drives (5)
9. Vector Control of Perm-Magnet Syn. Motor Drives (3)
10. Switched-Reluctance Motor (SRM) Drives (3)

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Continuation of Topics Discussed in the First Course

- Switch-Mode Converters Average Representation
- Magnetics – Transformers
- Feedback Controller Design
- Space Vector Representation for AC Machines
- Basic Calculations for Electromagnetic Torque
- PMAC Drives – Space Vector Based Steady State Operation
- Induction Motor Drives – Space Vector based Steady State Analysis

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Design Examples

- Induction Machine Initially Operating in Steady state
  - ◆ Load Torque Disturbance at t=0.1 s
  - ◆ Control Objective is to keep Speed Constant (design speed loop with a bandwidth of 25 rad/s and a phase margin of 60 degrees)
- Permanent Magnet AC (PMAC) Drives
- Switched-Reluctance Motor (SRM) Drives

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"TEST" INDUCTION MOTOR

Nameplate Data:

Power:	3 HP/2.4 kW
Voltage:	460 V (L-L, rms)
Frequency:	60 Hz
Phases:	3
Full Load Current:	4 A
Full-Load Speed:	1750 RPM
Full-Load Efficiency:	88.5 %
Power Factor:	80.0 %
Number of Poles:	4

Per-Phase Motor Circuit Parameters:

$$R_s = 1.77\,\Omega,\ R_r = 1.34\,\Omega,\ X_{\ell s} = 5.25\,\Omega \text{ (at 60 Hz)}$$

$$X_{\ell r} = 4.57\,\Omega \text{ (at 60 Hz)},\ X_m = 139.0\,\Omega \text{ (at 60 Hz)}$$

$$\text{Full-Load Slip} = 1.72\ \%,\ J_{eq} = 0.025\,kg \cdot m^2$$

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Chapter 2

Induction Machine Equations in Phase Quantities: Assisted by Space Vectors

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Sinusoidally-Distributed Stator Windings

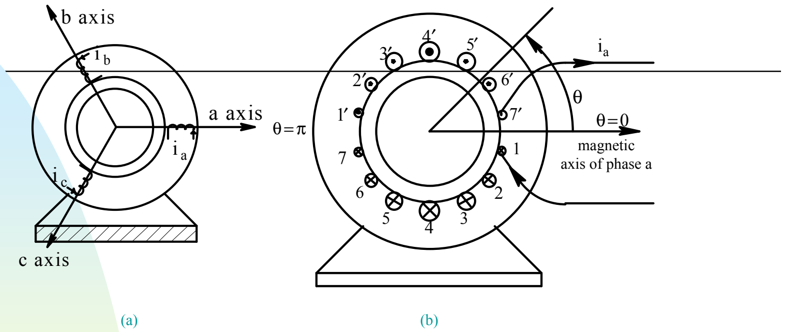

Figure 2-1 Stator windings.

(1) $n_s(\theta) = \frac{N_s}{2}\sin\theta \qquad 0 \le \theta \le \pi$

(2) $H_a(\theta) = \frac{N_s}{p\ell_g} i_a \cos\theta$

(3) $F_a(\theta) = \ell_g\, H_a(\theta) = \frac{N_s}{p} i_a \cos\theta$

(4) $B_a(\theta) = \mu_o H_a(\theta) = (\frac{\mu_o N_s}{p\ell_g}) i_a \cos\theta$

Three-Phase Sinusoidally-Distributed Windings

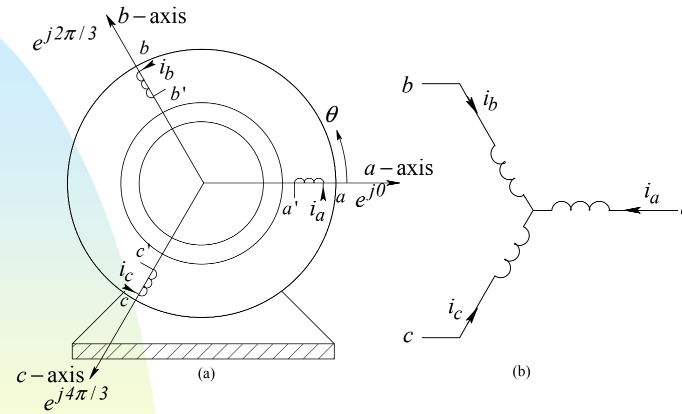

Figure 2-2 Three-phase windings.

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Single-Phase Magnetizing Inductance $L_{m,1-phase}$

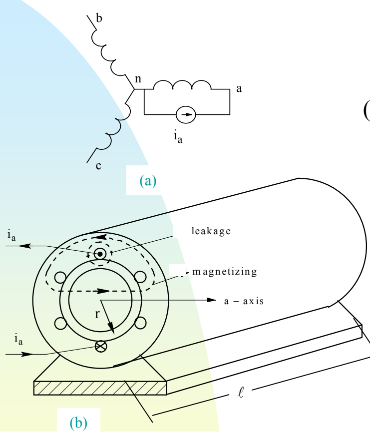

Figure 2-3 Single-phase magnetizing inductance and leakage inductance.

(1) $L_{s,self} = \left.\frac{\lambda_a}{i_a}\right|_{i_a\, only} = \underbrace{\frac{\lambda_{a,leakage}}{i_a}}_{L_{\ell s}} + \underbrace{\frac{\lambda_{a},magnetizing}{i_a}}_{L_{m,1-phase}}$

(2) $L_{s,self} = L_{\ell s} + L_{m,1-phase}$

(3) $L_{m,1-phase} = \frac{\pi\mu_o r\ell}{\ell_g}(\frac{N_s}{p})^2$

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Stator Mutual Inductance L_{mutual}

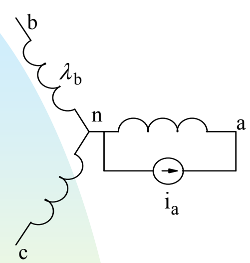

Figure 2-4 Mutual inductance .

(1) $L_{mutual} = \left.\frac{\lambda_b}{i_a}\right|_{i_b,\, i_c=0,\, rotor\ open}$

(2) $\lambda_{b,due\,to\,i_a} = \cos(120^o)\,\lambda_{a,magnetizing\ due\ to\ i_a}$

(3) $L_{mutual} = -\frac{1}{2} L_{m,1-phase}$

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Equivalent Windings in A Squirrel-Cage Rotor

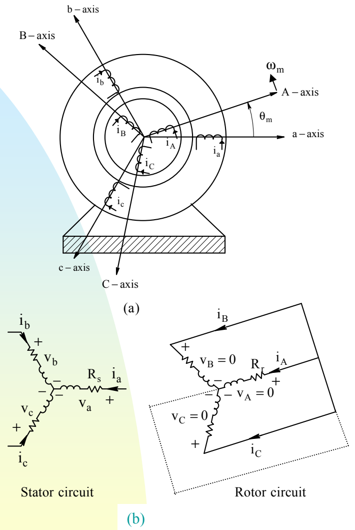

Figure 2-5 Rotor circuit represented by three-phase windings.

Stator

(1) $i_a(t) + i_b(t) + i_c(t) = 0$

(2) $L_m = \frac{3}{2} L_{m,1-phase}$

(3) $L_m = \frac{3}{2} \frac{\pi \mu_o r \ell}{\ell_g} (\frac{N_s}{p})^2$

(4) $L_s = L_{\ell s} + L_m$

Rotor

(1) $i_A(t) + i_B(t) + i_C(t) = 0$

(2) $L_m = \frac{3}{2} L_{m,1-phase}$

(3) $L_r = L_{\ell r} + L_m$

(4) $L_{aA} = L_{m,1-phase} \cdot \cos\theta_m$

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Review of Space Vectors

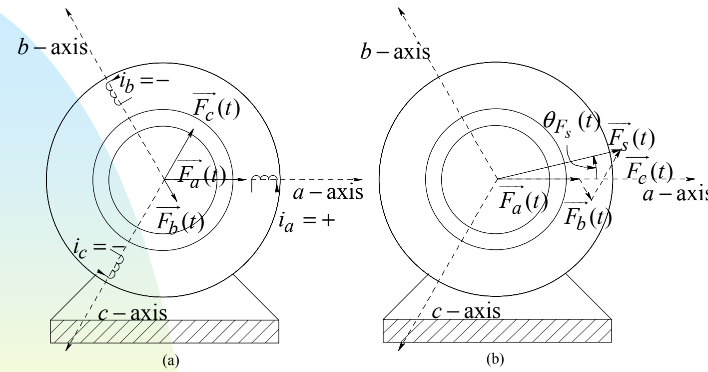

Figure 2-6 Space vector representation of mmf.

$$\vec{F}_s^a(t) = \vec{F}_a^a(t) + \vec{F}_b^a(t) + \vec{F}_c^a(t)$$

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Physical Interpretation of Current Space Vector

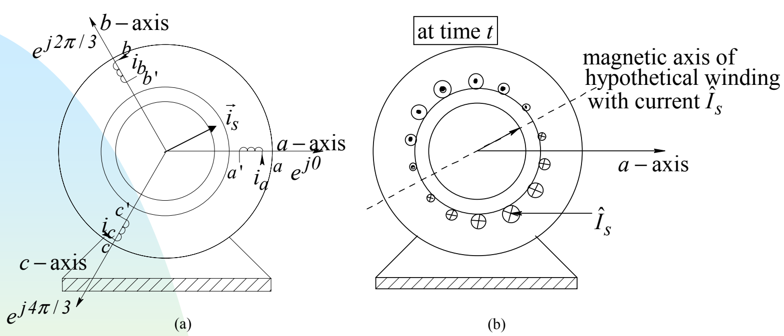

Figure 2-7 Physical interpretation of stator current space vector.

(1) $\vec{i}_s^{\,a}(t) = i_a(t)e^{j0} + i_b(t)e^{j2\pi/3} + i_c(t)e^{j4\pi/3} = \hat{I}_s(t)e^{j\theta_{i_s}(t)}$

(2) $\vec{F}_s^a(t) = (N_s / p)\vec{i}_s^{\,a}(t)$

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Voltage and Flux-Linkage Space Vectors

$$\vec{v}_s^{\,a}(t) = v_a(t)e^{j0} + v_b(t)e^{j2\pi/3} + v_c(t)e^{j4\pi/3} = \hat{V}_s(t)e^{j\theta_{v_s}(t)}$$

$$\vec{\lambda}_s^a(t) = \lambda_a(t)e^{j0} + \lambda_b(t)e^{j2\pi/3} + \lambda_c(t)e^{j4\pi/3} = \hat{\lambda}_s(t)e^{j\theta_{\lambda_s}(t)}$$

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Relationship between space vector and phasor in sinusoidal steady state

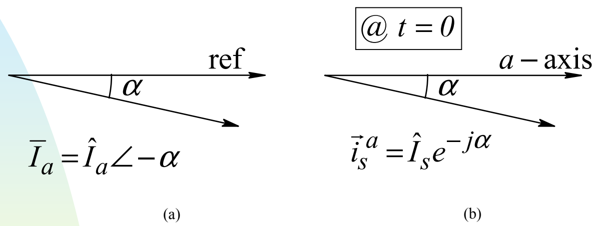

Figure 2-8 Relationship between space vector and phasor in sinusoidal steady state.

$$\vec{i}_s^{\,a}\Big|_{t=0} = \frac{3}{2}\bar{I}_a \qquad \hat{I}_s = \frac{3}{2}\hat{I}_a$$

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All stator space vectors are collinear (Rotor open-circuited)

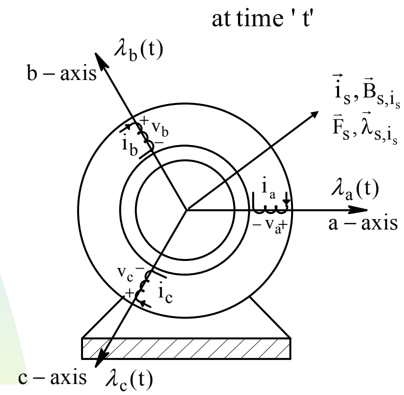

Figure 2-9 All stator space vectors are collinear (Rotor open-circuited).

$$\vec{\lambda}_{s,i_s}^{\,a}(t) = \underbrace{L_{\ell s}\vec{i}_s^{\,a}(t)}_{\text{due to leakage flux}} + \underbrace{L_m\vec{i}_s^{\,a}(t)}_{\text{due to magnetizing flux}} = L_s\vec{i}_s^{\,a}(t)$$

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All rotor space vectors are collinear (Stator open-circuited)

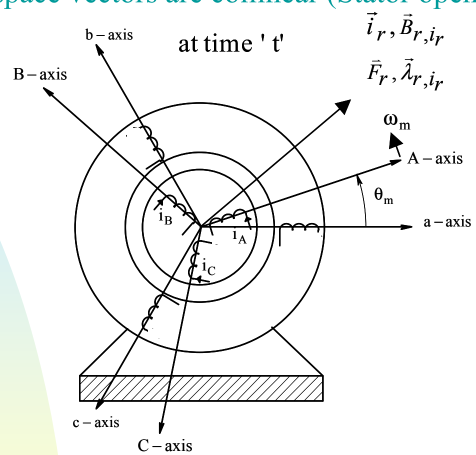

Figure 2-10 All rotor space vectors are collinear (Stator open-circuited).

$$\vec{\lambda}_{r,i_r}^{\,A}(t) = \underbrace{L_{\ell r}\vec{i}_r^{\,A}(t)}_{\text{due to leakage flux}} + \underbrace{L_m\vec{i}_r^{\,A}(t)}_{\text{due to magnetizing flux}} = L_r\vec{i}_r^{\,A}(t)$$

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Making the case for dq-axis analysis

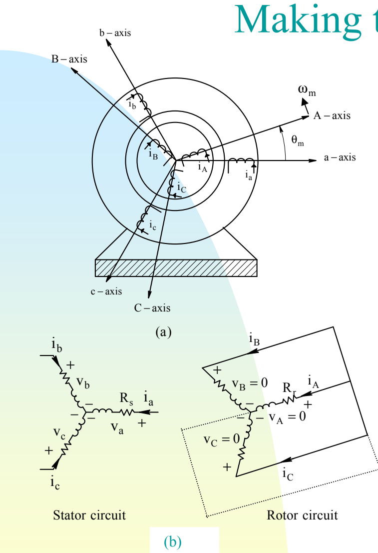

Figure 2-5 Rotor circuit represented by three-phase windings.

(1) $\vec{\lambda}_s^{\,a}(t) = L_s\vec{i}_s^{\,a}(t) + L_m\vec{i}_r^{\,a}$

(2) $\vec{i}_r^{\,a}(t) = \vec{i}_r^{\,A}(t)e^{j\theta_m}$

(3) $\vec{\lambda}_s^{\,a}(t) = L_s\vec{i}_s^{\,a}(t) + L_m\vec{i}_r^{\,A}(t)e^{j\theta_m}$

(4) $\vec{v}_s^{\,a}(t) = R_s\vec{i}_s^{\,a}(t) + \frac{d}{dt}\vec{\lambda}_s^{\,a}(t)$

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Chapter 3

Dynamic Analysis of Induction Machines in Terms of dq-Windings

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Representation of Stator MMF by Equivalent dq Windings

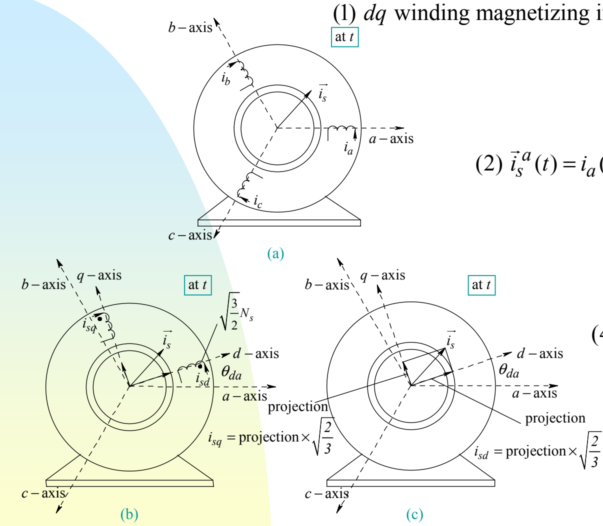

Figure 3-1 Representation of stator mmf by equivalent *dq* winding currents.

(1) *dq* winding magnetizing inductance $=(\sqrt{3/2})^2 L_{m,1-phase}$

$= (3/2) L_{m,1-phase}$

$= L_m$ (using Eq. 2-12)

(2) $\vec{i}_s^{\,a}(t) = i_a(t) + i_b(t)e^{j2\pi/3} + i_c(t)e^{j4\pi/3}$

(3) $\vec{F}_s^{\,a}(t) = \dfrac{N_s}{p}\vec{i}_s^{\,a}(t)$

(4) $\dfrac{\sqrt{3/2}N_s}{p}\left(i_{sd} + j i_{sq}\right) = \dfrac{N_s}{p}\vec{i}_s^{\,d}$

(5) $\left(i_{sd} + j i_{sq}\right) = \sqrt{\dfrac{2}{3}}\,\vec{i}_s^{\,d}$

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Representation of Rotor MMF by Equivalent dq Windings

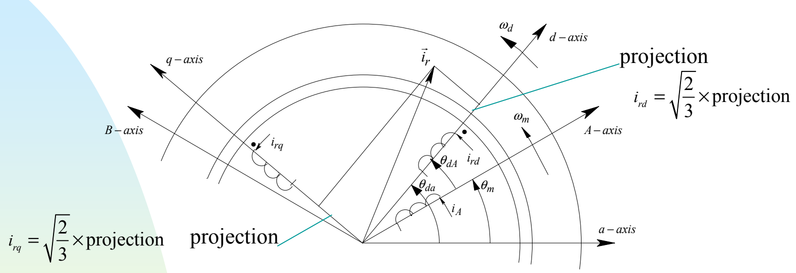

Figure 3-2 Representation of rotor mmf by equivalent *dq* winding currents.

$$\vec{i}_r^{\,A}(t) = i_A(t) + i_B(t)e^{j2\pi/3} + i_C(t)e^{j4\pi/3}$$

$$\vec{i}_r^{\,A}(t) = \frac{\vec{F}_r^{\,A}(t)}{N_s/p}$$

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Mutual Inductance between dq Windings on the Stator and the Rotor

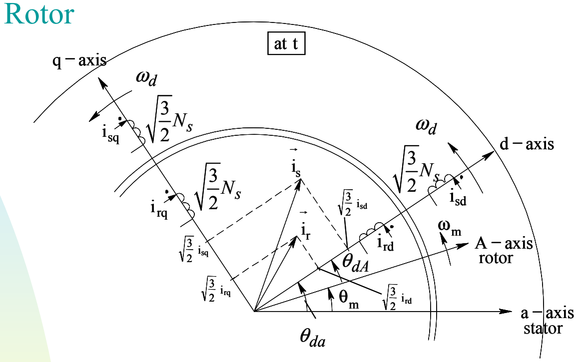

Figure 3-3 Stator and rotor representation by equivalent *dq* winding currents. The *dq* winding voltages are defined as positive at the dotted terminals. Note that the relative positions of the stator and the rotor current space vectors are not actual, rather only for definition purposes.

$\lambda_{sd} = L_s i_{sd} + L_m i_{rd}$ $\qquad$ $\lambda_{rd} = L_r i_{rd} + L_m i_{sd}$

$\lambda_{sq} = L_s i_{sq} + L_m i_{rq}$ $\qquad$ $\lambda_{rq} = L_r i_{rq} + L_m i_{sq}$

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Mathematical Relationship between dq and phase Winding Variables

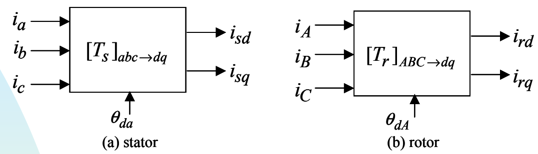

Figure 3-4 Transformation of phase quantities into *dq* winding quantities.

$$\begin{bmatrix} i_{sd}(t) \\ i_{sq}(t) \end{bmatrix} = \underbrace{\sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta_{da}) & \cos(\theta_{da} - \frac{2\pi}{3}) & \cos(\theta_{da} - \frac{4\pi}{3}) \\ -\sin(\theta_{da}) & -\sin(\theta_{da} - \frac{2\pi}{3}) & -\sin(\theta_{da} - \frac{4\pi}{3}) \end{bmatrix}}_{[T_s]_{abc \to dq}} \begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix}$$

$$\begin{bmatrix} i_{rd}(t) \\ i_{rq}(t) \end{bmatrix} = \underbrace{\sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta_{dA}) & \cos(\theta_{dA} - \frac{2\pi}{3}) & \cos(\theta_{dA} - \frac{4\pi}{3}) \\ -\sin(\theta_{dA}) & -\sin(\theta_{dA} - \frac{2\pi}{3}) & -\sin(\theta_{dA} - \frac{4\pi}{3}) \end{bmatrix}}_{[T_r]_{ABC \to dq}} \begin{bmatrix} i_A(t) \\ i_B(t) \\ i_C(t) \end{bmatrix}$$

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Derivation of Stator Voltages in dq Windings

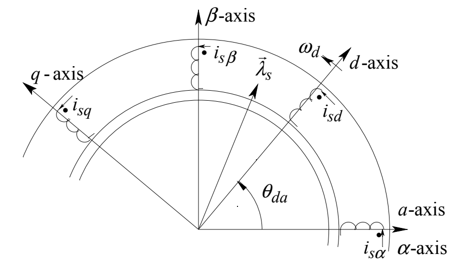

Figure 3-5 Stator $\alpha\beta$ and dq equivalent windings.

(1) $v_{s\alpha} = R_s i_{s\alpha} + \frac{d}{dt}\lambda_{s\alpha}$

(2) $v_{s\beta} = R_s i_{s\beta} + \frac{d}{dt}\lambda_{s\beta}$

(3) $\vec{v}^{\,\alpha}_{s\_\alpha\beta} = v_{s\alpha} + j v_{s\beta}$; etc.

(4) $\vec{v}^{\,\alpha}_{s\_\alpha\beta} = R_s \vec{i}^{\,\alpha}_{s\_\alpha\beta} + \frac{d}{dt}\vec{\lambda}^{\alpha}_{s\_\alpha\beta}$

(5) $\vec{v}_{s\_dq} = v_{sd} + j v_{sq}$; etc.

(6) $\vec{v}^{\,\alpha}_{s\_\alpha\beta} = \vec{v}_{s\_dq} \cdot e^{j\theta_{da}}$; etc.

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Derivation of Stator Voltages in dq Windings (continued)

(1) $\vec{v}^{\,\alpha}_{s\_\alpha\beta} = R_s \vec{i}^{\,\alpha}_{s\_\alpha\beta} + \frac{d}{dt}\vec{\lambda}^{\alpha}_{s\_\alpha\beta}$

(2) $\vec{v}_{s\_dq} = R_s \vec{i}_{s\_dq} + \frac{d}{dt}\vec{\lambda}_{s\_dq} + j\omega_d \vec{\lambda}_{s\_dq}$

(3) $v_{sd} = R_s i_{sd} + \frac{d}{dt}\lambda_{sd} - \omega_d \lambda_{sq}$

(4) $v_{sq} = R_s i_{sq} + \frac{d}{dt}\lambda_{sq} + \omega_d \lambda_{sd}$

(5) $\begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix} = R_s \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \lambda_{sd} \\ \lambda_{sq} \end{bmatrix} + \omega_d \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{[M_{rotate}]} \begin{bmatrix} \lambda_{sd} \\ \lambda_{sq} \end{bmatrix}$

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Derivation of Rotor Voltages in dq Windings

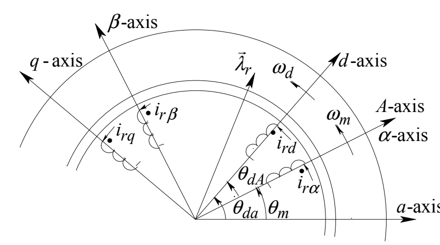

Figure 3-6 Rotor $\alpha\beta$ and dq equivalent windings.

(1) $v_{rd} = R_r i_{rd} + \frac{d}{dt}\lambda_{rd} - \omega_{dA}\lambda_{rq}$

(2) $v_{rq} = R_r i_{rq} + \frac{d}{dt}\lambda_{rq} + \omega_{dA}\lambda_{rd}$

(3) $\omega_{dA} = \omega_d - \omega_m$

(4) $\omega_m = (p/2)\,\omega_{mech}$

(5) $\begin{bmatrix} v_{rd} \\ v_{rq} \end{bmatrix} = R_r \begin{bmatrix} i_{rd} \\ i_{rq} \end{bmatrix} + \frac{d}{dt}\begin{bmatrix} \lambda_{rd} \\ \lambda_{rq} \end{bmatrix} + \omega_{dA} \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{[M_{rotate}]} \begin{bmatrix} \lambda_{rd} \\ \lambda_{rq} \end{bmatrix}$

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Obtaining Flux Linkages: Voltages as Inputs

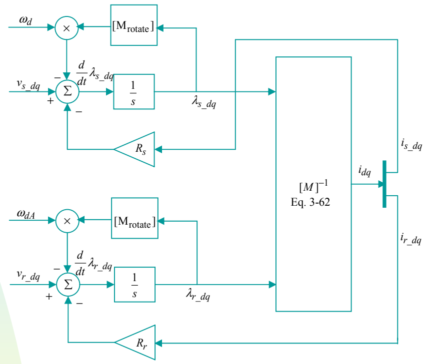

Figure 3-7 Calculating *dq* winding flux linkages and currents.

$$\frac{d}{dt}\begin{bmatrix}\lambda_{sd}\\ \lambda_{sq}\end{bmatrix}=\begin{bmatrix}v_{sd}\\ v_{sq}\end{bmatrix}-R_s\begin{bmatrix}i_{sd}\\ i_{sq}\end{bmatrix}-\omega_d\underbrace{\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}}_{[\mathrm{M_{rotate}}]}\begin{bmatrix}\lambda_{sd}\\ \lambda_{sq}\end{bmatrix}$$

$$\frac{d}{dt}[\lambda_{s\_dq}]=[v_{s\_dq}]-R_s[i_{s\_dq}]-\omega_d\,[\mathrm{M_{rotate}}][\lambda_{s\_dq}]$$

$$\frac{d}{dt}\begin{bmatrix}\lambda_{rd}\\ \lambda_{rq}\end{bmatrix}=\begin{bmatrix}v_{rd}\\ v_{rq}\end{bmatrix}-R_r\begin{bmatrix}i_{rd}\\ i_{rq}\end{bmatrix}-\omega_{dA}\underbrace{\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}}_{[\mathrm{M_{rotate}}]}\begin{bmatrix}\lambda_{rd}\\ \lambda_{rq}\end{bmatrix}$$

$$\frac{d}{dt}[\lambda_{r\_dq}]=[v_{r\_dq}]-R_r[i_{r\_dq}]-\omega_{dA}\,[\mathrm{M_{rotate}}][\lambda_{r\_dq}]$$

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Electromagnetic Torque on the Rotor d-Axis

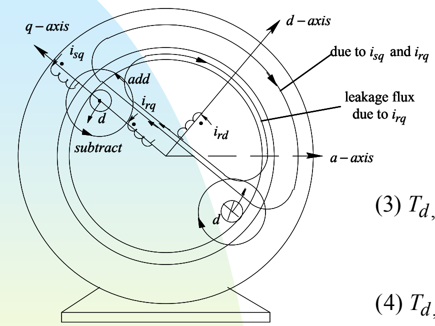

Figure 3-8 Torque on the rotor *d*-axis.

(1) $\hat{B}_{rq}=\frac{\mu_0}{\ell_g}\underbrace{\left(\frac{\sqrt{3/2}N_s}{p}\right)(i_{sq}+\frac{L_r}{L_m}i_{rq})}_{mmf}$

(2) $T_{d,rotor}=\frac{p}{2}\left(\pi\frac{\sqrt{3/2}N_s}{p}r\ell\hat{B}_{rq}\right)i_{rd}$

(3) $T_{d,rotor}=\frac{p}{2}\left(\pi\frac{\mu_0}{\ell_g}r\ell\right)\left(\frac{\sqrt{3/2}N_s}{p}\right)^2(i_{sq}+\frac{L_r}{L_m}i_{rq})i_{rd}$

(4) $T_{d,rotor}=\frac{p}{2}\underbrace{\left(\frac{3}{2}\pi\frac{\mu_0}{\ell_g}r\ell\left(\frac{N_s}{p}\right)^2\right)}_{L_m}(i_{sq}+\frac{L_r}{L_m}i_{rq})i_{rd}$

(5) $T_{d,rotor}=\frac{p}{2}\underbrace{(L_m i_{sq}+L_r i_{rq})}_{\lambda_{rq}}i_{rd}=\frac{p}{2}\lambda_{rq}i_{rd}$

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Electromagnetic Torque on the Rotor q-Axis

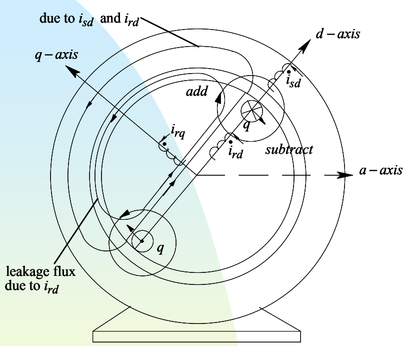

Figure 3-9 Torque on the rotor *q*-axis.

$$T_{q,rotor}=-\frac{p}{2}\underbrace{(L_m i_{sq}+L_r i_{rq})}_{\lambda_{rd}}i_{rq}=-\frac{p}{2}\lambda_{rd}i_{rq}$$

Net Electromagnetic Torque

(1) $T_{em}=T_{d,rotor}+T_{q,rotor}$

(2) $T_{em}=\frac{p}{2}(\lambda_{rq}i_{rd}-\lambda_{rd}i_{rq})$

(3) $T_{em}=\frac{p}{2}L_m(i_{sq}i_{rd}-i_{sd}i_{rq})$

(4) $\frac{d}{dt}\omega_{mech}=\frac{T_{em}-T_L}{J_{eq}}$

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Equivalent d and q Axis Equivalent Circuits

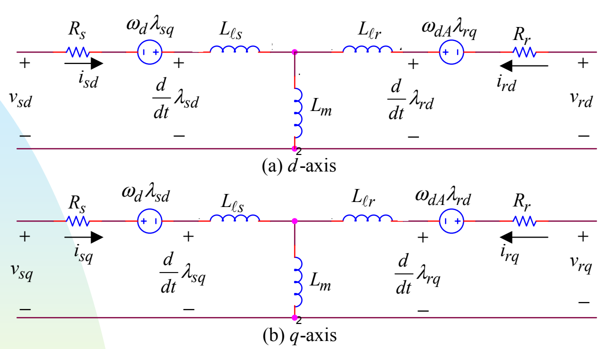

Figure 3-10 *dq*-winding equivalent circuits.

$$v_{sd}=R_s i_{sd}-\omega_d\lambda_{sq}+L_{\ell s}\frac{d}{dt}i_{sd}+L_m\frac{d}{dt}(i_{sd}+i_{rd})$$

$$v_{sq}=R_s i_{sq}+\omega_d\lambda_{sd}+L_{\ell s}\frac{d}{dt}i_{sq}+L_m\frac{d}{dt}(i_{sq}+i_{rq})$$

$$\underbrace{v_{rd}}_{=0}=R_r i_{rd}-\omega_{dA}\lambda_{rq}+L_{\ell r}\frac{d}{dt}i_{rd}+L_m\frac{d}{dt}(i_{sd}+i_{rd})$$

$$\underbrace{v_{rq}}_{=0}=R_r i_{rq}+\omega_{dA}\lambda_{rd}+L_{\ell r}\frac{d}{dt}i_{rq}+L_m\frac{d}{dt}(i_{sq}+i_{rq})$$

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Per-Phase Equivalent Circuit

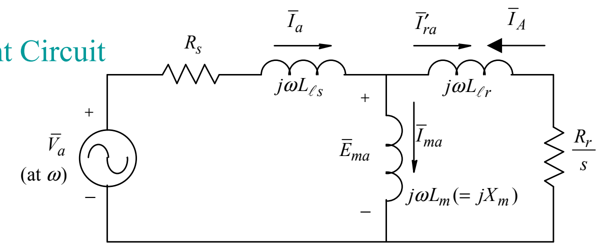

Figure 3-11 Per-phase equivalent circuit in steady state.

(1) $\vec{v}_{s\_dq} = R_s \vec{i}_{s\_dq} + j\omega_{syn}\vec{\lambda}_{s\_dq}$

(2) $\vec{v}_{s\_dq} = R_s \vec{i}_{s\_dq} + j\omega_{syn} L_{\ell s}\vec{i}_{s\_dq} + j\omega_{syn} L_m(\vec{i}_{s\_dq} + \vec{i}_{r\_dq})$

(3) $\bar{V}_a = R_s \bar{I}_a + j\omega_{syn} L_{\ell s}\bar{I}_a + j\omega_{syn} L_m(\bar{I}_a + \bar{I}_A)$

(4) $0 = \frac{R_r}{s}\vec{i}_{r\_dq} + j\omega_{syn}\vec{\lambda}_{r\_dq}$

(5) $0 = \frac{R_r}{s}\vec{i}_{r\_dq} + j\omega_{syn} L_{\ell r}\vec{i}_{r\_dq} + j\omega_{syn} L_m(\vec{i}_{s\_dq} + \vec{i}_{r\_dq})$

(6) $0 = \frac{R_r}{s}\bar{I}_a + j\omega_{syn} L_{\ell r}\bar{I}_A + j\omega_{syn} L_m(\bar{I}_a + \bar{I}_A)$

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Obtaining Currents from Flux Linkages

$$\begin{bmatrix}\lambda_{sd}\\ \lambda_{rd}\end{bmatrix} = \underbrace{\begin{bmatrix} L_s & L_m \\ L_m & L_r\end{bmatrix}}_{[L]}\begin{bmatrix} i_{sd}\\ i_{rd}\end{bmatrix}$$

$$\begin{bmatrix}\lambda_{sq}\\ \lambda_{rq}\end{bmatrix} = \underbrace{\begin{bmatrix} L_s & L_m \\ L_m & L_r\end{bmatrix}}_{[L]}\begin{bmatrix} i_{sq}\\ i_{rq}\end{bmatrix}$$

$$\begin{bmatrix}\lambda_{sd}\\ \lambda_{sq}\\ \lambda_{rd}\\ \lambda_{rq}\end{bmatrix} = \underbrace{\begin{bmatrix} L_s & 0 & L_m & 0 \\ 0 & L_s & 0 & L_m \\ L_m & 0 & L_r & 0 \\ 0 & L_m & 0 & L_r\end{bmatrix}}_{[M]}\begin{bmatrix} i_{sd}\\ i_{sq}\\ i_{rd}\\ i_{rq}\end{bmatrix} \qquad \begin{bmatrix} i_{sd}\\ i_{sq}\\ i_{rd}\\ i_{rq}\end{bmatrix} = [M]^{-1}\begin{bmatrix}\lambda_{sd}\\ \lambda_{sq}\\ \lambda_{rd}\\ \lambda_{rq}\end{bmatrix}$$

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Induction Motor Model

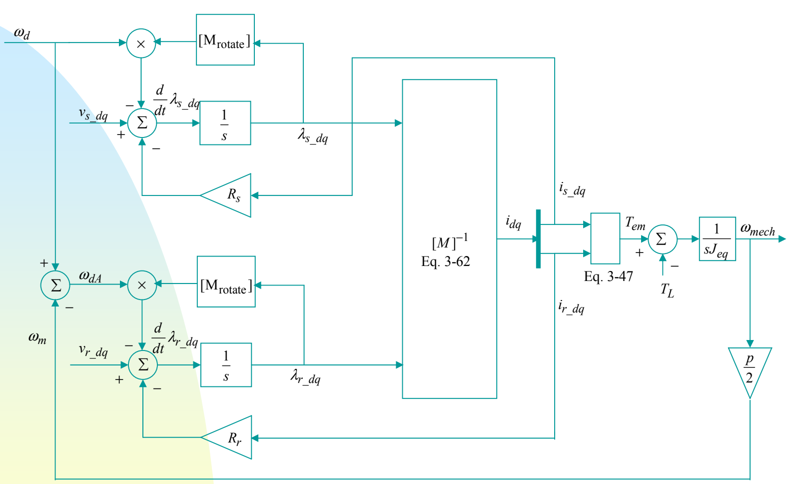

Figure 3-12 Induction motor model in terms of dq windings.

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Calculation of Steady State Initial Conditions Using Phasors

$$\bar{I}_a = \hat{I}_a \angle \theta_i \quad \Rightarrow \quad \vec{i}_s(0) = \underbrace{\frac{3}{2}\hat{I}_a}_{\hat{I}_s} e^{j\theta_i}$$

$$i_{sd}(0) = \sqrt{\frac{2}{3}} \times projection\ of\ \vec{i}_s(0)\ on\ d\text{-}axis = \sqrt{\frac{2}{3}}\underbrace{\left(\frac{3}{2}\hat{I}\right)}_{\hat{I}_s}\cos(\theta_i)$$

$$i_{sq}(0) = \sqrt{\frac{2}{3}} \times projection\ of\ \vec{i}_s(0)\ on\ q\text{-}axis = \sqrt{\frac{2}{3}}\underbrace{\left(\frac{3}{2}\hat{I}\right)}_{\hat{I}_s}\sin(\theta_i)$$

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Calculation of Steady State Initial Conditions Using Voltage Equations

(1) $v_{sd} = R_s i_{sd} - \omega_{syn}\lambda_{sq}$

(2) $v_{sq} = R_s i_{sq} + \omega_{syn}\lambda_{sd}$

(3) $0 = R_r i_{rd} - s\omega_{syn}\lambda_{rq}$

(4) $0 = R_r i_{rq} + s\omega_{syn}\lambda_{rd}$

$$\begin{bmatrix} v_{sd} \\ v_{sq} \\ 0 \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} R_s & -\omega_{syn}L_s & 0 & -\omega_{syn}L_m \\ \omega_{syn}L_s & R_s & \omega_{syn}L_m & 0 \\ 0 & -s\omega_{syn}L_m & R_r & -s\omega_{syn}L_r \\ s\omega_{syn}L_m & 0 & s\omega_{syn}L_r & R_r \end{bmatrix}}_{[A]} \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix}$$

$$\begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix} = [A]^{-1} \begin{bmatrix} v_{sd} \\ v_{sq} \\ 0 \\ 0 \end{bmatrix}$$

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Simlink-based dq-Axis Simulation of Induction Motor

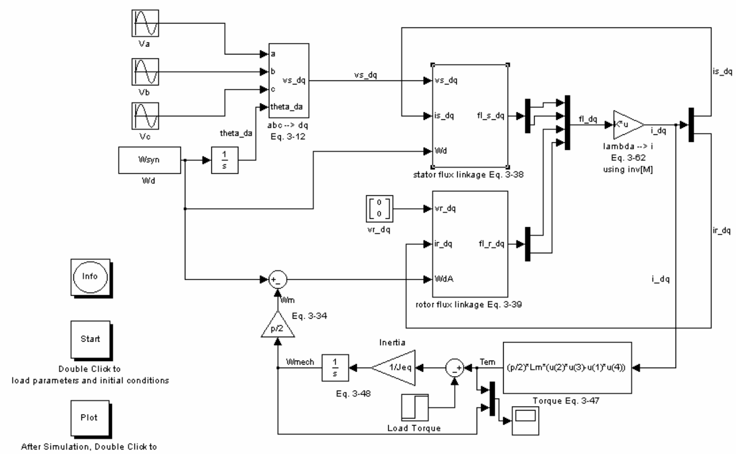

Figure 3-13 Simulation of Example 3-3; File Name EX3_3.mdl

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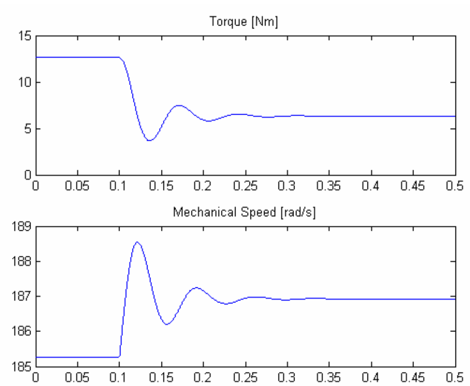
Simulation Results



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Chapter 4

Vector Control of Induction-Motor Drives: A Qualitative Examination

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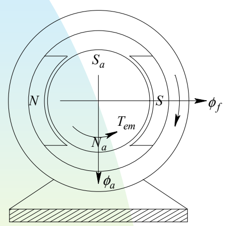
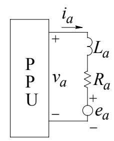
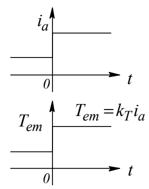
DC Motor Drive







$T_{em} = k_T\ i_a$

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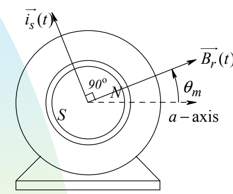
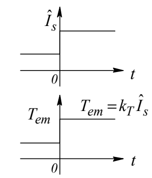
Brushless DC Motor Drive





$$T_{em} = k_T\ \hat{I}_S$$

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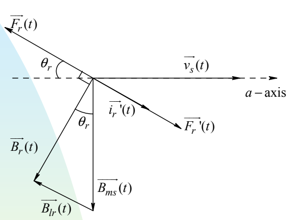
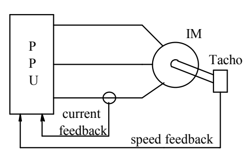
Vector-Controlled Induction Motor Drive





$\vec{B}_r(t)$ perpendicuar to $\vec{F}_r'(t)$ and $\vec{F}_r(t)$

$T_{em} = k_T\ \hat{I}_r'$ (keeping $\hat{B}_r$ constant)

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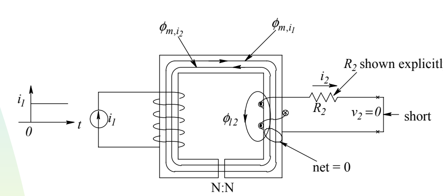
Analogy to a Current-Excited Transformer With a Shorted Secondary



$$\lambda_2(0^+) = \lambda_2(0^-) = 0$$

$$\phi_{m,i_1}(0^+) = \phi_{m,i_2}(0^+) + \phi_{\ell 2}(0^+)$$

$$i_2(0^+) = \frac{L_m}{L_2}\ i_1(0^+)$$

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Analogy to a Current-Excited Transformer With a Shorted Secondary

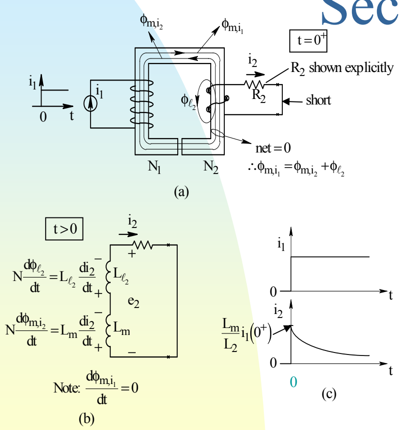

Figure 4-5 Analogy of A Current-Excited Transformer with a Short-Circuited Secondary; $N_1 = N_2$

$$L_2 = L_m + L_{\ell 2}$$

$$i_2(0^+) = \frac{L_m}{L_2} i_1(0^+)$$

$$\tau_2 = \frac{L_2}{R_2}$$

$$i_2(t) = i_2(0^+) e^{-t/\tau_2}$$

Using the Transformer Equivalent Circuit

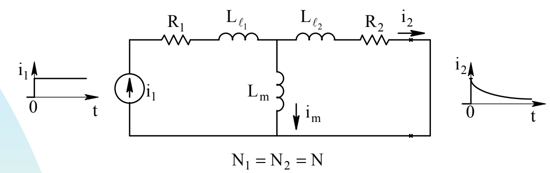

Figure 4-6 Equivalent-Circuit Representation of the Current-Excited Transformer with a Short-Circuited Secondary.

$$i_2(0^+) = \frac{L_m}{L_m + L_{\ell 2}} i_1(0^+) = \frac{L_m}{L_2} i_1(0^+)$$

$$i_m(0^+) = \frac{L_{\ell 2}}{L_m + L_{\ell 2}} i_1(0^+) = \frac{L_{\ell 2}}{L_2} i_1(0^+)$$

$$i_2(t) = i_2(0^+) e^{-t/\tau_2}$$

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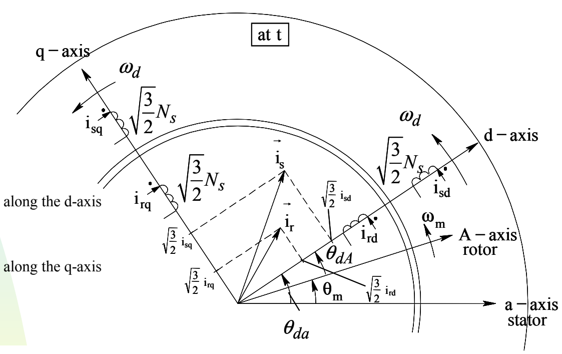
d- and q- Axis Winding Representation

$i_{sd} = \sqrt{\frac{2}{3}}$ the projection of $\vec{i}_s(t)$ vector along the d-axis

$i_{sq} = \sqrt{\frac{2}{3}}$ the projection of $\vec{i}_s(t)$ vector along the q-axis



Stator and rotor representation by equivalent *dq* winding currents. The *dq* winding voltages are defined as positive at the dotted terminals. Note that the relative positions of the stator and the rotor current space vectors are not actual, rather only for definition purposes.

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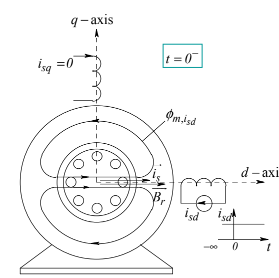
Initial Flux Buildup Prior to $t = 0^-$



$$i_a(0^-) = \hat{I}_{m,rated} \text{ and } i_b(0^-) = i_c(0^-) = -\frac{1}{2}\hat{I}_{m,rated}$$

$$i_{sd}(0^-) = \sqrt{\frac{2}{3}}\hat{I}_{ms,rated} = \sqrt{\frac{2}{3}}\left(\frac{3}{2}\hat{I}_{m,rated}\right) = \sqrt{\frac{3}{2}}\hat{I}_{m,rated}$$

$$i_{sq} = 0$$

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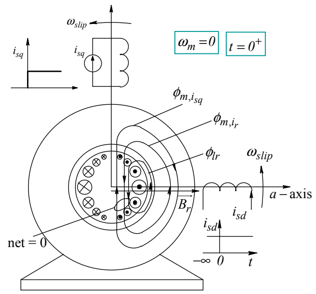
Step Change in Torque at $t = 0^-$



- $\omega_m = 0$
- i_{sd} unchanged
- Step-change in i_{sq}
  - $\Phi_{q,net} = 0$
  - $\phi_{m,i_{sq}}(0^+) = \phi_{m,i_r}(0^+) + \phi_{\ell r}(0^+)$
  - $T_{em} \ \alpha \ \hat{B}_r, \dfrac{L_m}{L_r} i_{sq}$

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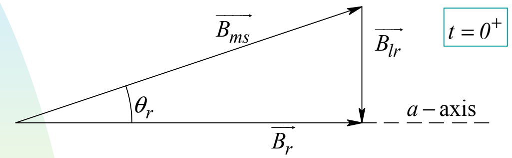
Flux Densities at $t = 0^+$



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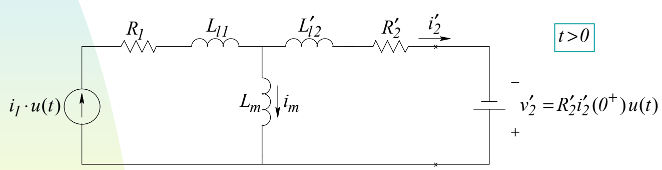
Transformer Analogy – Voltage Needed to Prevent the Decay of Secondary Current



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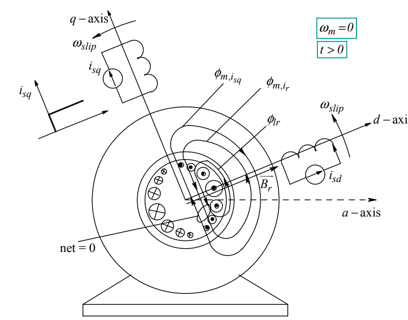
Currents and Fluxes at Sometime Later t > 0, Blocked Rotor



$$\omega_{slip} \ \alpha \ \frac{R'_r, (L_m/L_r) i_{sq}}{\hat{B}_r}$$

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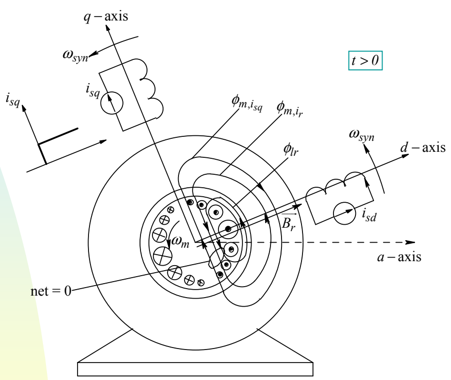
Vector-Controlled Condition With a Rotor Speed ω_m



$$\omega_{syn} = \omega_m + \omega_{slip}$$

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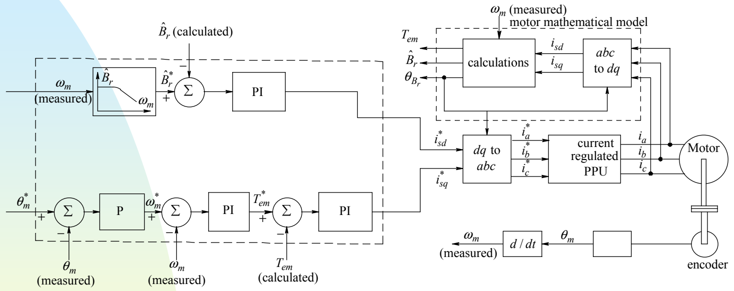
Torque, Speed, and Position Control



$$\omega_{syn}(t) = \omega_m(t) + \omega_{slip}(t)$$

$$\theta_{B_r}(t) = 0 + \int_0^t \omega_{syn}(\tau) \cdot d\tau$$

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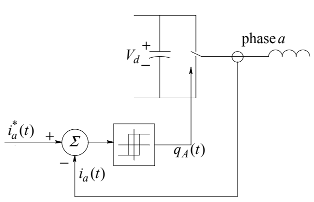
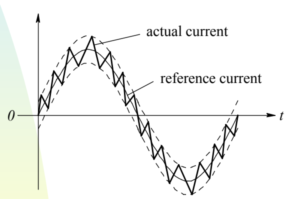
Power-Processing Unit (PPU)



(a)



(b)

Figure 4-14 (a) Block diagram representation of hysteresis current control; (b) current waveform.

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Chapter 5

Mathematical Description of Vector Control

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Motor Model with the d-Axis Aligned with the Rotor Flux Linkage Axis

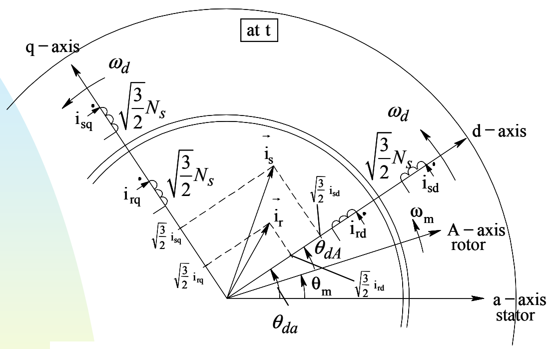

Figure 5-1 Stator and rotor mmf representation by equivalent *dq* winding currents. The *d*-axis is aligned with $\vec{\lambda}_r$.

$$\lambda_{rq}(t)=0 \qquad \frac{d}{dt}\lambda_{rq}(t)=0 \qquad i_{rq}=-\frac{L_m}{L_r}i_{sq}$$

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Dynamic Circuits with the d-Axis Aligned with the Rotor Flux Linkage Axis

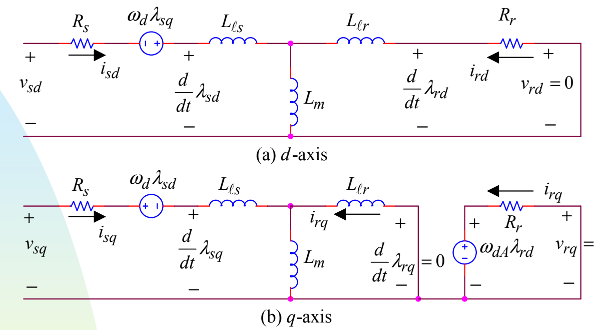

Figure 5-2 Dynamic circuits with the *d*-axis aligned with $\vec{\lambda}_r$.

Calculation of ω_{dA} :

$$\omega_{dA}=-R_r\frac{i_{rq}}{\lambda_{rd}}=\frac{L_m}{\tau_r\lambda_{rd}}i_{sq}$$

Calculation of Torque T_{em} :

$$T_{em}=-\frac{p}{2}\lambda_{rd}i_{rq}=\frac{p}{2}\lambda_{rd}\left(\frac{L_m}{L_r}i_{sq}\right)$$

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D-Axis Rotor Flux Dynamics

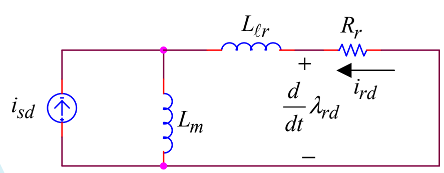

Figure 5-3 The *d*-axis circuit simplified with a current excitation.

(1) $i_{rd}(s)=-\frac{sL_m}{R_r+sL_r}i_{sd}(s)$

(2) $\lambda_{rd}=L_r i_{rd}+L_m i_{sd}$

(3) $\lambda_{rd}(s)=\frac{L_m}{(1+s\tau_r)}i_{sd}(s)$

(4) $\frac{d}{dt}\lambda_{rd}+\frac{\lambda_{rd}}{\tau_r}=\frac{L_m}{\tau_r}i_{sd}$

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Motor Model

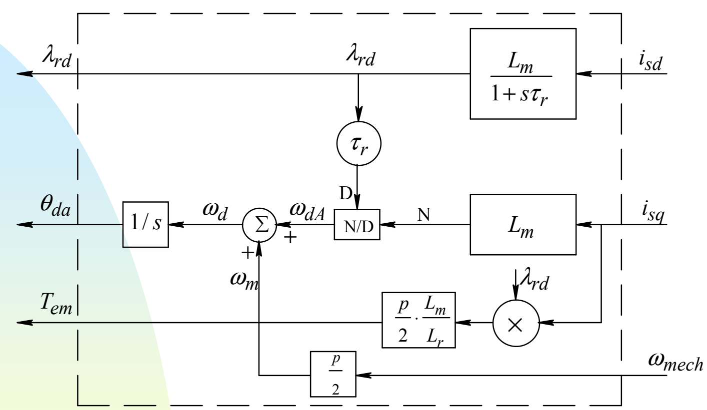

Figure 5-4 Motor model with d-axis aligned with $\vec{\lambda}_r$.

$$\theta_{da}(t)=0+\int_0^t\omega_d(\tau)\,d\tau$$

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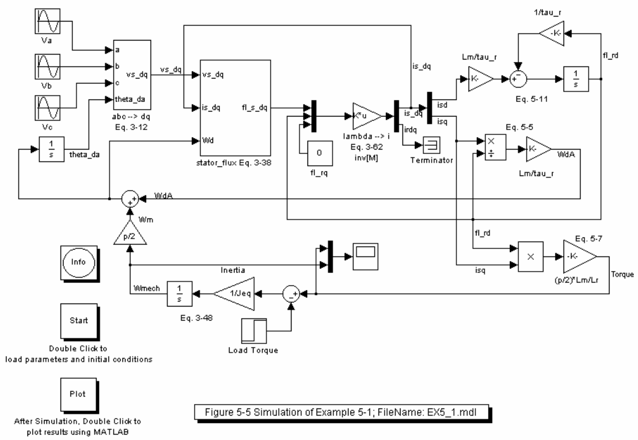
Simulation with d-Axis Aligned with the Rotor Flux Linkage (line-fed machine; achieving vector control is not an objective)



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Simulation Results match that of Example 3-3

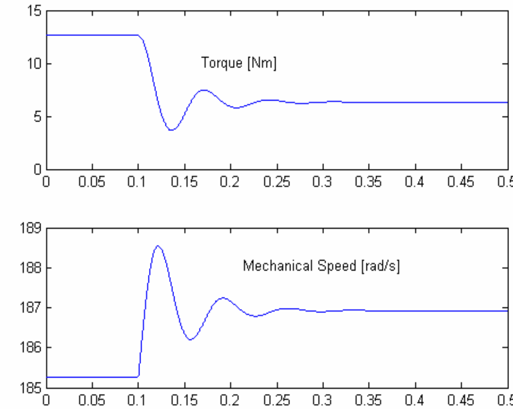

Figure 5-6 Results of Example 5-1.

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Vector-Controlled Induction Motor with a CR-PWM Inverter

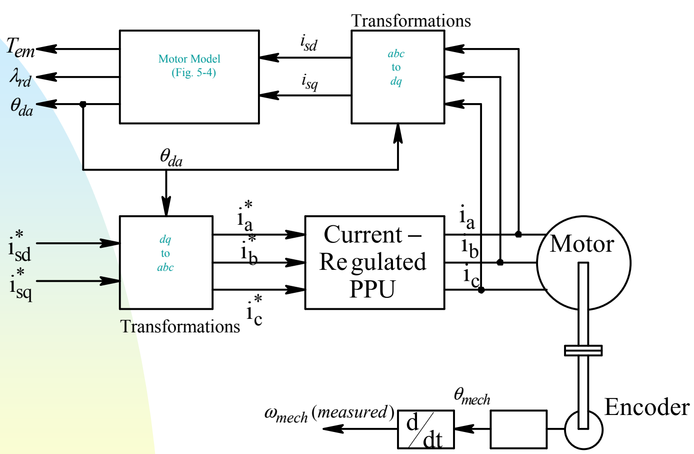

Figure 5-7 Vector-controlled induction motor with a CR-PWM Inverter.

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Speed and Position Loops for Vector Control

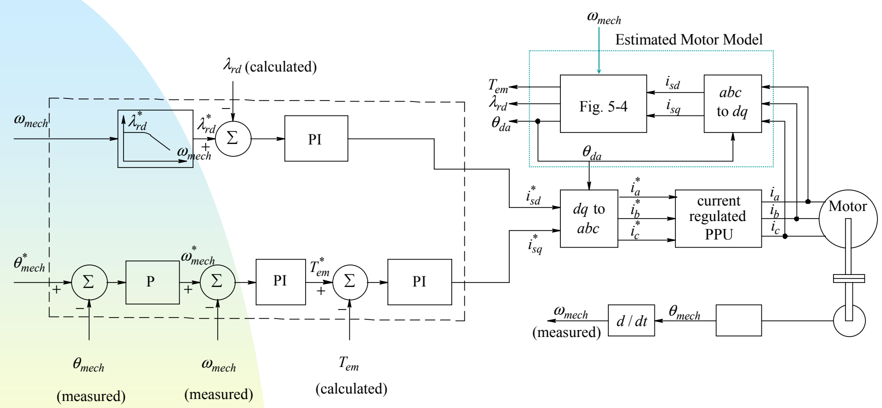

Figure 5-8 Vector controlled induction motor drive with a current-regulated PPU.

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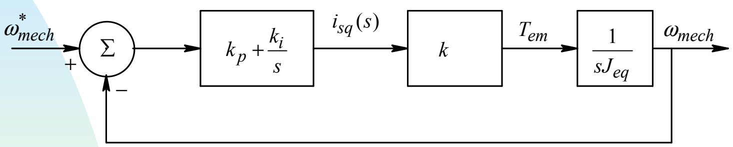
Design of Speed Loop



$$\lambda_{rd} = L_m i_{sd}$$

$$T_{em} = \underbrace{\frac{p}{2}\frac{L_m^2}{L_r} i^*_{sd}}_{k} i_{sq}$$

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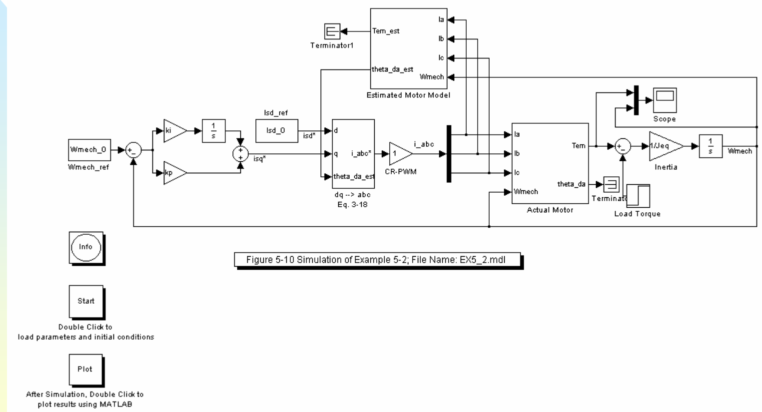
Simulation of CR-PWM Vector Controlled Drive



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Simulation Results of a Vector Controlled Induction Motor Drive

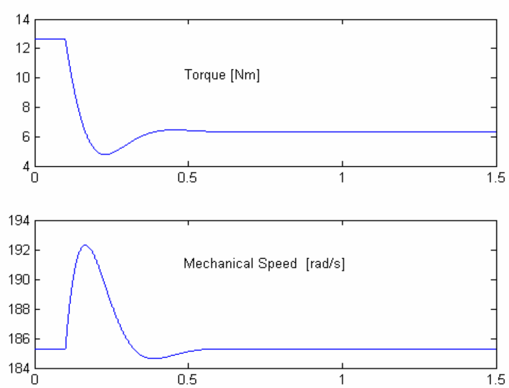

Figure 5-11 Simulation results of Example 5-2.

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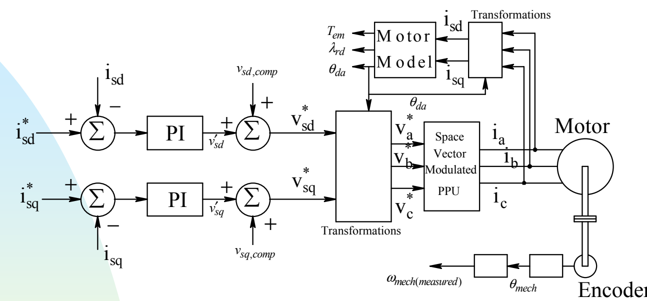
Calculation of Stator Voltages in Vector Control



(1) $\sigma = 1 - \frac{L_m^2}{L_s L_r}$

(2) $\lambda_{sd} = \sigma L_s i_{sd} + \frac{L_m}{L_r}\lambda_{rd}$

(3) $\lambda_{sq} = \sigma L_s i_{sq}$

(4) $v_{sd} = \underbrace{R_s i_{sd} + \sigma L_s \frac{d}{dt} i_{sd}}_{v'_{sd}} + \underbrace{\frac{L_m}{L_r}\frac{d}{dt}\lambda_{rd} - \omega_d \sigma L_s i_{sq}}_{v_{sd,comp}}$

(5) $v_{sq} = \underbrace{R_s i_{sq} + \sigma L_s \frac{d}{dt} i_{sq}}_{v'_{sq}} + \underbrace{\omega_d \frac{L_m}{L_r}\lambda_{rd} + \omega_d \sigma L_s i_{sd}}_{v_{sq,comp}}$

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Design of the Current-Loop Controller

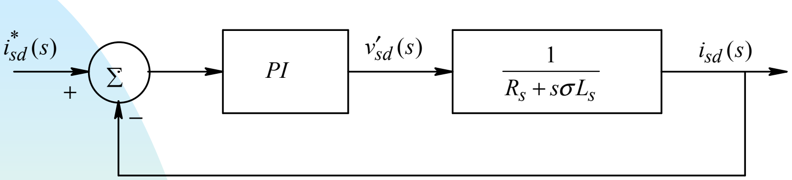

Figure 5-13 Design of the current-loop controller.

(1) $v'_{sd} = R_s i_{sd} + \sigma L_s \frac{d}{dt} i_{sd}$

(2) $v'_{sq} = R_s i_{sq} + \sigma L_s \frac{d}{dt} i_{sq}$

(3) $i_{sd}(s) = \frac{1}{R_s + s\sigma L_s} v'_{sd}(s)$

(4) $i_{sq}(s) = \frac{1}{R_s + s\sigma L_s} v'_{sq}(s)$

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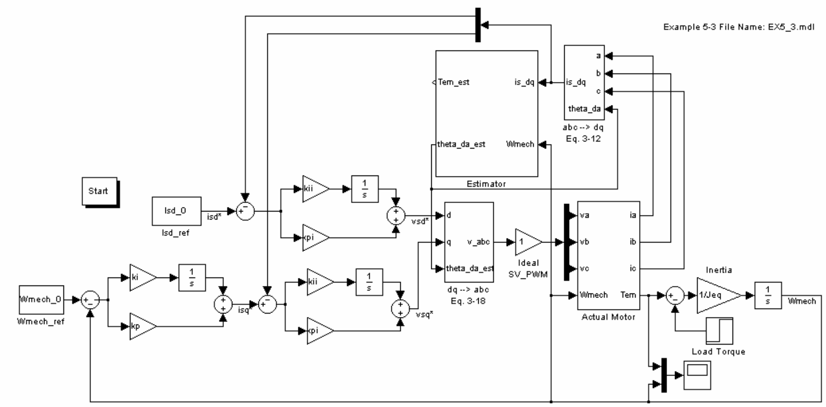
Simulation of Vector Controlled Drive with supplied Voltages



Simulation Results of Vector Controlled Drive with supplied Voltages

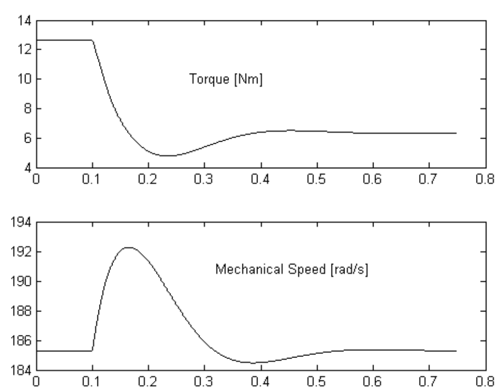


Figure 5-15 Simulation results of Example 5-3.

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Chapter 6

Detuning Effects in Induction Motor Vector Control

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Initial conditions

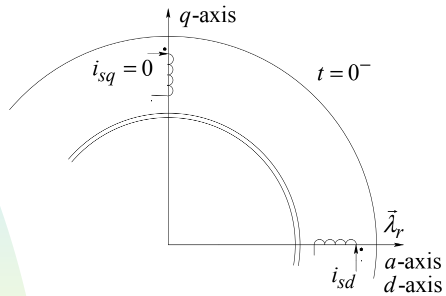

Figure 6-1 dq windings at $t = 0^-$.

$t = 0^-$

$i_{sd} = i_{sd}^*$

$\theta_{da} = \theta_{da,est} = 0$

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Effect of Detuning due to Incorrect Rotor Time Constant

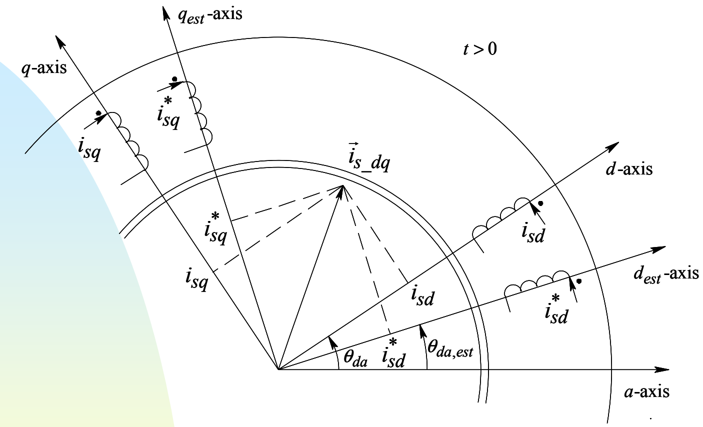

Figure 6-2 dq windings at $t > 0$; drawn for $k_\tau < 1$.

(1) $k_\tau = \dfrac{\tau_r}{\tau_{r,est}}$ (2) $\vec{i}_{s\_dq}^{\,d,est} = i_{sd}^* + j i_{sq}^*$ (4) $\theta_{err} = \theta_{da} - \theta_{da,est}$

(3) $\vec{i}_{s\_dq}^{\,d} = \vec{i}_{s\_dq}^{\,d,est}\, e^{-j(\theta_{da} - \theta_{da,est})} = \vec{i}_{s\_dq}^{\,d,est}\, e^{-j(\theta_{err})}$

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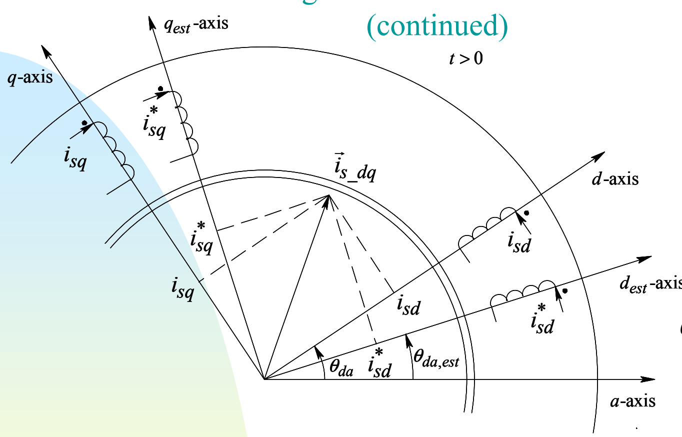
Effect of Detuning due to Incorrect Rotor Time Constant (continued)



$\theta_{err} = \theta_{da} - \theta_{da,est}$

Figure 6-2 dq windings at $t > 0$; drawn for $k_\tau < 1$.

(1) $\vec{i}_{s\_dq}^{\,d} = \vec{i}_{s\_dq}^{\,d,est}\, e^{-j(\theta_{da} - \theta_{da,est})} = \vec{i}_{s\_dq}^{\,d,est} e^{-j(\theta_{err})}$ (3) $i_{sd} = i_{sd}^* \cos\theta_{err} + i_{sq}^* \sin\theta_{err}$

(2) $\vec{i}_{s\_dq}^{\,d,est} = i_{sd}^* + j i_{sq}^*$ (4) $i_{sq} = i_{sq}^* \cos\theta_{err} - i_{sd}^* \sin\theta_{err}$

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Estimated Motor Model (Rotor Blocked)

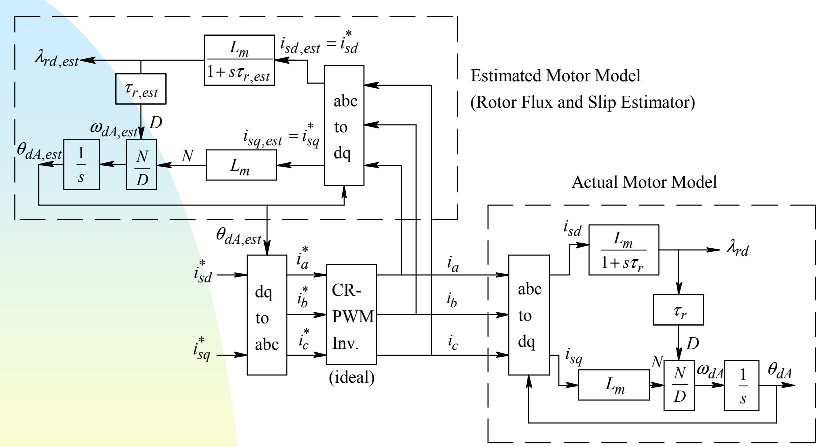

Figure 6-3 Actual and the estimated motor models (blocked-rotor).

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Simulation of Vector Control with Estimated Motor Parameters

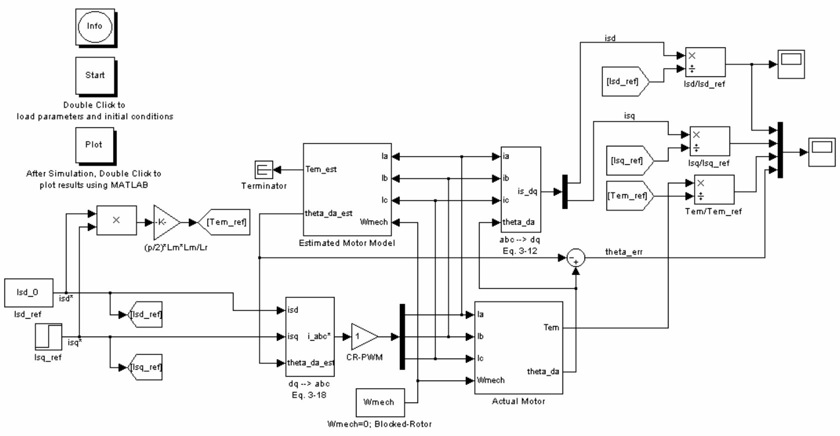

Figure 6-4 Simulation of Example 6-1; File Name: EX6_1.mdl

Effect of Detuning in Dynamic and Steady States

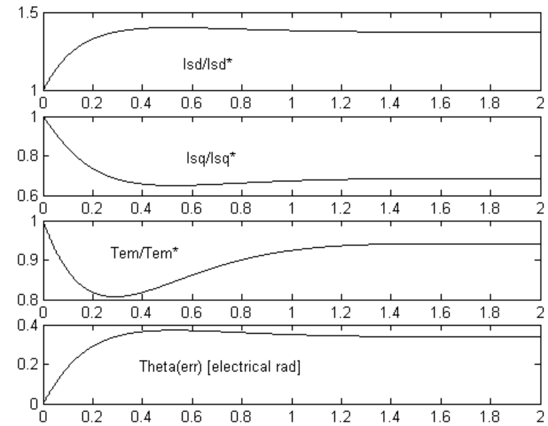

Figure 6-5 Simulation results of Example 6-1.

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Calculations of Steady State Errors

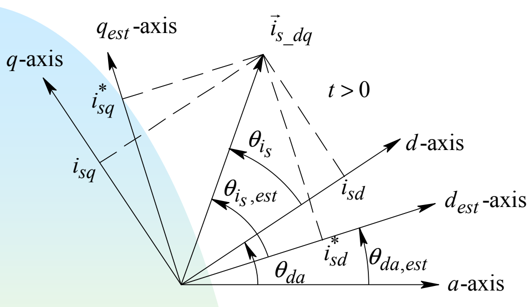

Figure 6-6 dq windings at $t > 0$; drawn for $k_\tau < 1$.

$$\sqrt{i_{sd}^2 + i_{sq}^2} = \sqrt{i_{sd}^{*2} + i_{sq}^{*2}} = \hat{I}_{s\_dq}$$

$$\omega_{dA} = \frac{1}{\tau_r}\frac{i_{sq}}{i_{sd}}$$

$$\omega_{dA,est} = \frac{1}{\tau_{r,est}}\frac{i_{sq}^*}{i_{sd}^*}$$

$$\underbrace{\frac{1}{\tau_{r,est}}\frac{i_{sq}^*}{i_{sd}^*}}_{\omega_{dA,est}} = \underbrace{\frac{1}{\tau_r}\frac{i_{sq}}{i_{sd}}}_{\omega_{dA}}$$

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Calculations of Steady State Errors (continued)

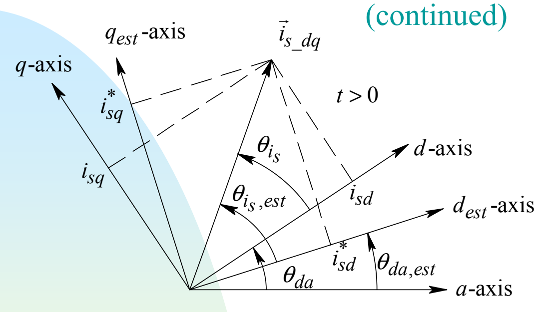

Figure 6-6 dq windings at $t > 0$; drawn for $k_\tau < 1$.

$$m = \frac{i_{sq}^*}{i_{sd}^*}$$

$$\frac{i_{sd}}{i_{sd}^*} = \sqrt{\frac{1+m^2}{1+k_\tau^2 \cdot m^2}}$$

$$\frac{i_{sq}}{i_{sq}^*} = k_\tau \frac{i_{sd}}{i_{sd}^*}$$

$$\theta_{err} = \theta_{i_s,est} - \theta_{i_s}$$

$$\theta_{err} = \tan^{-1}(m) - \tan^{-1}(k_\tau \cdot m)$$

$$\frac{T_{em}}{T_{em}^*} = k_\tau \frac{1+m^2}{1+(k_\tau \cdot m)^2}$$

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Chapter 7

Space-Vector Pulse-Width-Modulated (SV-PWM) Inverters

Advantages

- Full Utilization of the DC Bus Voltage
- Same simplicity as the Carrier-Modulated PWM
- Applicable in Vector Control, DTC and V/f Control

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Synthesis of Stator Voltage Space Vector

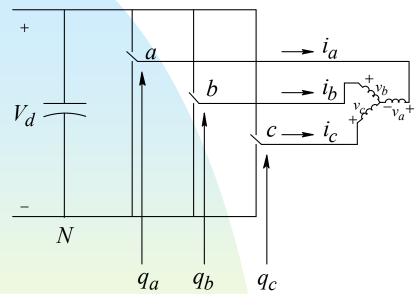

Figure 7-1 Switch-mode inverter.

(1) $\vec{v}_s^{\,a}(t) = v_a(t)e^{j0} + v_b(t)e^{j2\pi/3} + v_c(t)e^{j4\pi/3}$

(2) $v_a = v_{aN} + v_N; \quad v_b = v_{bN} + v_N; \quad v_c = v_{cN} + v_N$

(3) $e^{j0} + e^{j2\pi/3} + e^{j4\pi/3} = 0$

(4) $\vec{v}_s^{\,a}(t) = v_{aN}e^{j0} + v_{bN}e^{j2\pi/3} + v_{cN}e^{j4\pi/3}$

(5) $\vec{v}_s^{\,a}(t) = V_d(q_a e^{j0} + q_b e^{j2\pi/3} + q_c e^{j4\pi/3})$

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Basic Voltage Vectors

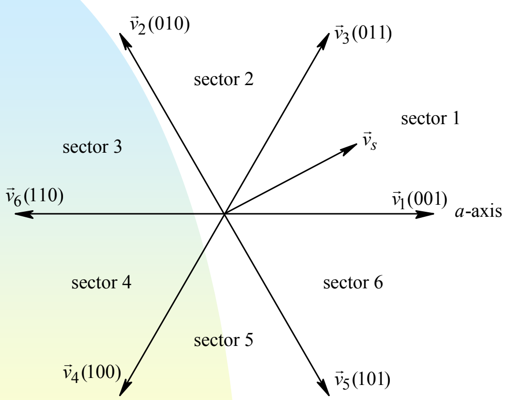

Figure 7-2 Basic voltage vectors ($\vec{v}_0$ and $\vec{v}_7$ not shown).

$\vec{v}_s^{\,a}(000) = \vec{v}_0 = 0$

$\vec{v}_s^{\,a}(001) = \vec{v}_1 = V_d e^{j0}$

$\vec{v}_s^{\,a}(010) = \vec{v}_2 = V_d e^{j2\pi/3}$

$\vec{v}_s^{\,a}(011) = \vec{v}_3 = V_d e^{j\pi/3}$

$\vec{v}_s^{\,a}(100) = \vec{v}_4 = V_d e^{j4\pi/3}$

$\vec{v}_s^{\,a}(101) = \vec{v}_5 = V_d e^{j5\pi/3}$

$\vec{v}_s^{\,a}(110) = \vec{v}_6 = V_d e^{j\pi}$

$\vec{v}_s^{\,a}(111) = \vec{v}_7 = 0$

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Synthesis of Voltage Vector in Sector 1

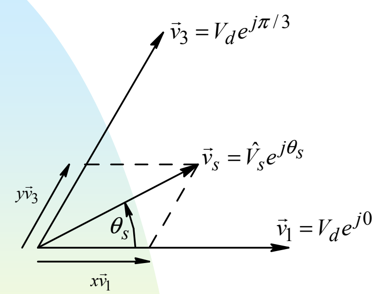

Figure 7-3 Voltage vector in sector 1.

(1) $\vec{v}_s^{\,a} = \frac{1}{T_s}[xT_s\vec{v}_1 + yT_s\vec{v}_3 + zT_s \cdot 0]$

(2) $\vec{v}_s^{\,a} = x\vec{v}_1 + y\vec{v}_3$

(3) $x + y + z = 1$

(4) $\hat{V}_s e^{j\theta_s} = xV_d e^{j0} + yV_d e^{j\pi/3}$

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Synthesis using Carrier-Modulated PWM

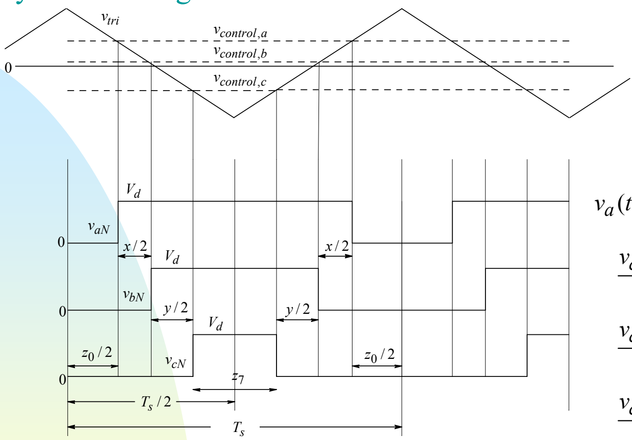

Figure 7-4 Waveforms in sector 1; $z = z_0 + z_7$.

$$v_a(t) + v_b(t) + v_c(t) = 0$$

$$\frac{v_{control,a}}{\hat{V}_{tri}} = \frac{v_a - v_k}{V_d/2}$$

$$\frac{v_{control,b}}{\hat{V}_{tri}} = \frac{v_b - v_k}{V_d/2}$$

$$\frac{v_{control,c}}{\hat{V}_{tri}} = \frac{v_c - v_k}{V_d/2}$$

$$v_k = \frac{\max(v_a, v_b, v_c) + \min(v_a, v_b, v_c)}{2}$$

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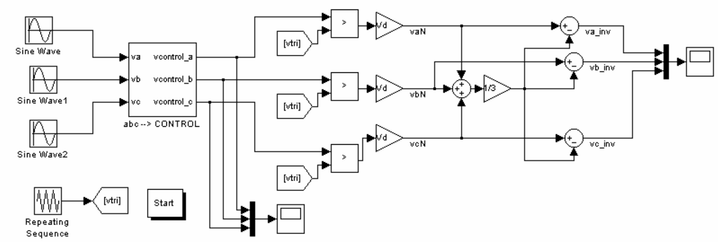
Synthesis of Space Vector using Carrier-Modulated PWM in Simulink



Control Waveforms for Carrier Pulse-Width-Modulation

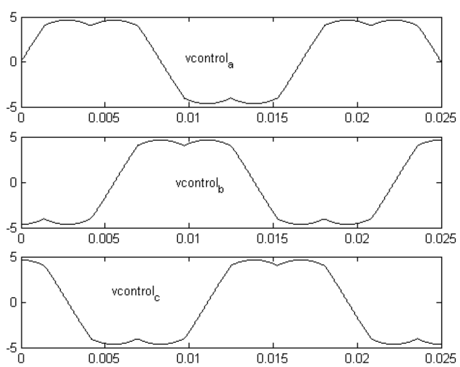


Figure 7-6 Simulation results of Example 7-1.

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Limit on the Amplitude of the Stator Voltage Space Vector

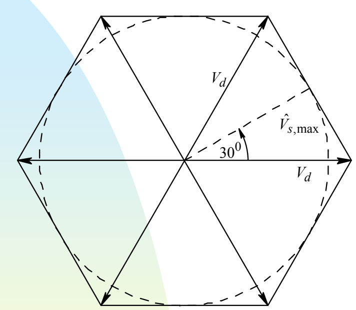

Figure 7-7 Limit on amplitude $\hat{V}_s$.

(1) $\vec{v}^{\,a}_{s,\max}(t) = \hat{V}_{s,\max} e^{j\omega_{syn} t}$

(2) $\hat{V}_{s,\max} = V_d \cos(\frac{60^0}{2}) = \frac{\sqrt{3}}{2} V_d$

(3) $\hat{V}_{phase,\max} = \frac{2}{3}\hat{V}_{s,\max} = \frac{V_d}{\sqrt{3}}$

(4) $V_{LL,\max}(rms) = \sqrt{3}\,\frac{\hat{V}_{phase,\max}}{\sqrt{2}} = \frac{V_d}{\sqrt{2}} = 0.707 V_d$

(5) $V_{LL,\max}(rms) = \frac{\sqrt{3}}{2\sqrt{2}} V_d = 0.612 V_d$ (sinusoidal PWM)

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Chapter 8

Direct Torque Control (DTC) and Encoder-less Operation of Induction Motors

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DTC System Overview

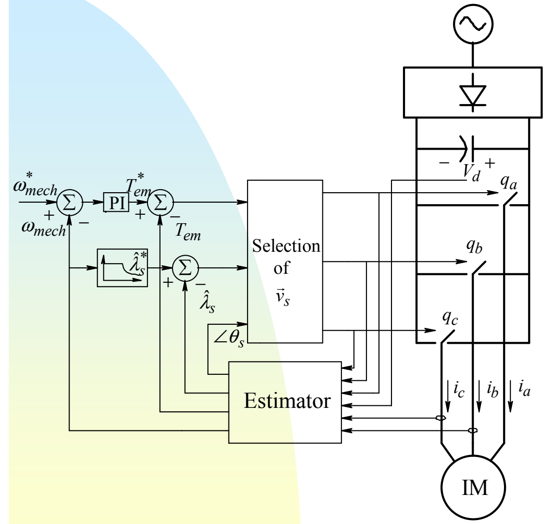

Figure 8-1 Block diagram of DTC.

Measured Inputs: Stator Voltages and Currents

Estimated Outputs: 1) Torque, 2) Mechanical Speed, 3) Stator Flux Amplitude and 4) its angle

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Principle of DTC Operation

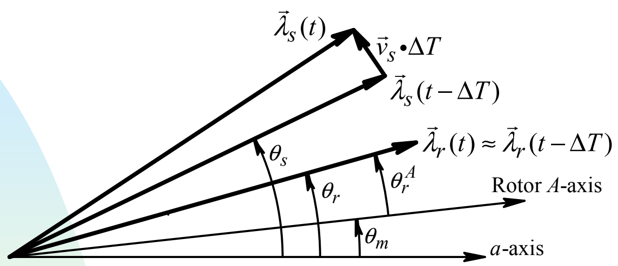

Figure 8-2 Changing the position of stator flux-linkage vector.

$$T_{em} = \frac{p}{2}\frac{L_m}{L_\sigma^2}\hat{\lambda}_s\hat{\lambda}_r \sin\theta_{sr}$$

$$\theta_{sr} = \theta_s - \theta_r$$

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Calculation of Stator Flux:

$$\vec{v}_s = R_s\vec{i}_s + \frac{d}{dt}\vec{\lambda}_s \Rightarrow \vec{\lambda}_s(t) = \vec{\lambda}_s(t-\Delta T) + \int_{t-\Delta T}^{t}(\vec{v}_s - R_s\vec{i}_s)\cdot d\tau = \hat{\lambda}_s e^{j\theta_s}$$

Calculation of Rotor Flux:

$$\vec{\lambda}_r = \frac{L_r}{L_m}(\vec{\lambda}_s - \sigma L_s \vec{i}_s) = \hat{\lambda}_r e^{j\theta_r}$$

where $\sigma = 1 - \frac{L_m^2}{L_s L_r}$

Estimating Torque:

$$T_{em} = \frac{p}{2}\text{Im}(\vec{\lambda}_s^{conj}\vec{i}_s)$$

Estimating Mechanical Speed:

$$\omega_r = \frac{d}{dt}\theta_r = \frac{\theta_r(t) - \theta_r(t-\Delta T_\omega)}{\Delta T_\omega}$$

$$\omega_{slip} = \frac{2}{p}\left(\frac{3}{2}R_r\frac{T_{em}}{\hat{\lambda}_r^2}\right)$$

$$\omega_m = \omega_r - \omega_{slip}$$

$$\omega_{mech} = (2/p)\,\omega_m$$

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Inverter Basic Vectors and Sectors

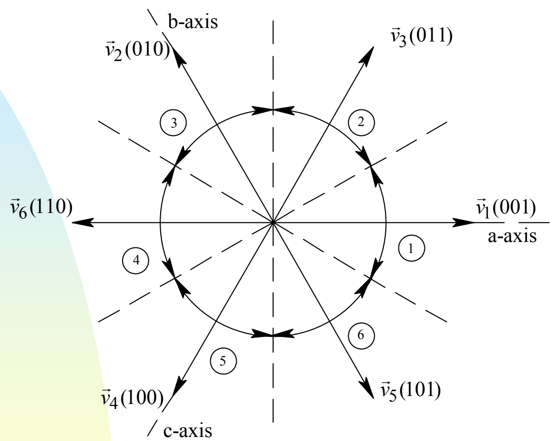

Figure 8-3 Inverter basic vectors and sectors.

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Stator Voltage Vector Selection in Sector 1

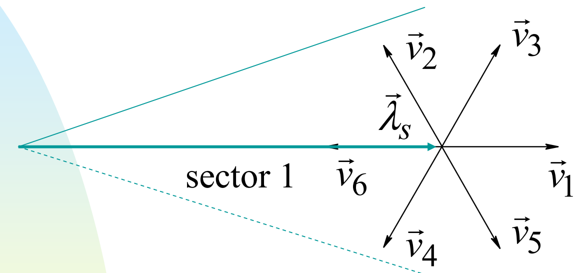

Figure 8-4 Stator voltage vector selection in sector 1.

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Selection of the Stator Voltage Space Vector

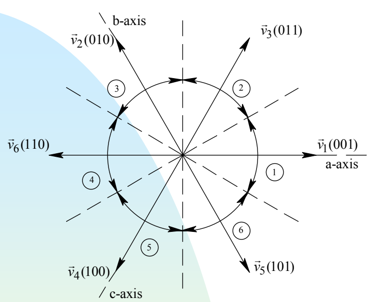

Figure 8-3 Inverter basic vectors and sectors.

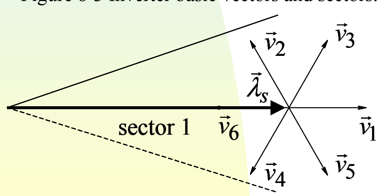

Figure 8-4 Stator voltage vector selection in sector 1.

Effect of Voltage Vector on the Stator Flux-Linkage Vector in Sector 1.

$\vec{v}_s$	T_{em}	$\hat{\lambda}_s$
$\vec{v}_3$	increase	increase
$\vec{v}_2$	increase	decrease
$\vec{v}_4$	decrease	decrease
$\vec{v}_5$	decrease	increase

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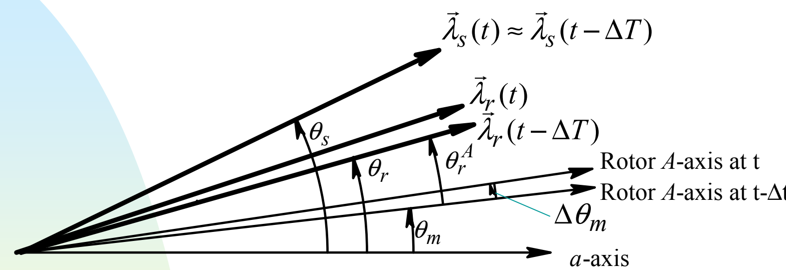
Effect of Zero Stator Voltage Space Vector



$\Delta\theta_s \simeq 0$

$\Delta\theta_r^A \simeq 0$

$\Delta\theta_r = \Delta\theta_m + \Delta\theta_r^A \simeq \Delta\theta_m$

$\sin\theta_{sr} \approx (\theta_s - \theta_r)$

$T_{em} \simeq k(\theta_s - \theta_r)$

$\Delta T_{em} \simeq -k(\Delta\theta_m)$

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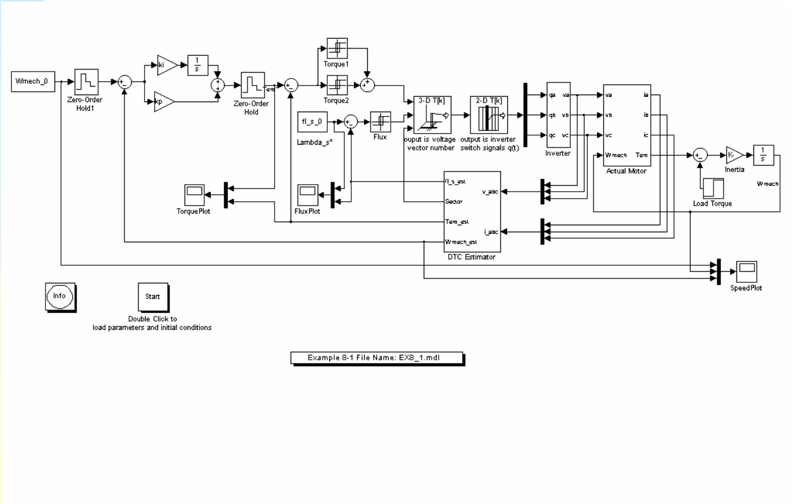
DTC in Simulink



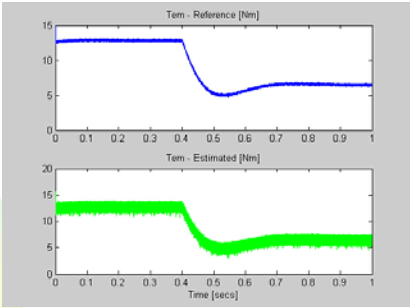


Fig. 2 Torque Waveforms.

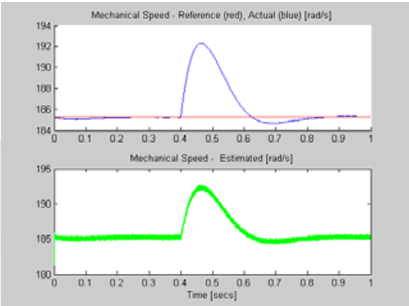


Fig. 3 Speed Waveforms.

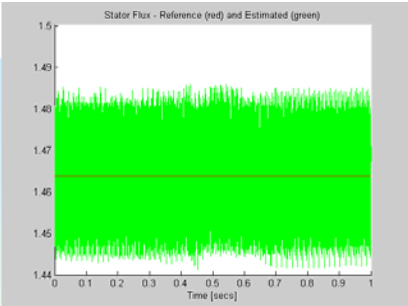


Fig. 4 Stator Flux.

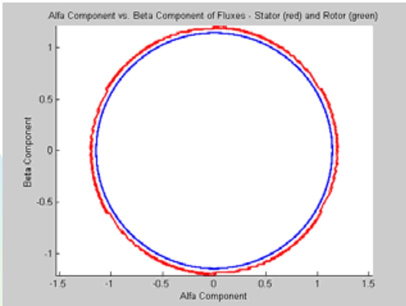


Fig. 5 Stator and Rotor Fluxes.

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Chapter 9

Vector Control of Permanent-Magnet Synchronous-Motor Drives

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Non-Salient Permanent-Magnet Synchronous Motor

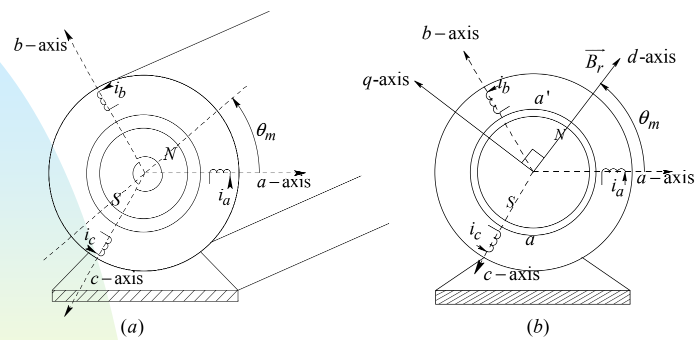

Figure 9-1 Permanent-magnet synchronous machine (shown with p=2).

$$\lambda_{sd} = L_s i_{sd} + \lambda_{fd} \qquad \lambda_{sq} = L_s i_{sq}$$

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Non-Salient Permanent-Magnet Synchronous Motor (Continued)

$$v_{sd} = R_s i_{sd} + \frac{d}{dt}\lambda_{sd} - \omega_m \lambda_{sq}$$

$$v_{sq} = R_s i_{sq} + \frac{d}{dt}\lambda_{sq} + \omega_m \lambda_{sd}$$

$$T_{em} = \frac{p}{2}(\lambda_{sd} i_{sq} - \lambda_{sq} i_{sd}) \qquad T_{em} = \frac{p}{2}[(L_s i_{sd} + \lambda_{fd}) i_{sq} - L_s i_{sq} i_{sd}] = \frac{p}{2}\lambda_{fd} i_{sq}$$

$$\omega_m = \frac{p}{2}\omega_{mech}$$

$$\frac{d}{dt}\omega_{mech} = \frac{T_{em} - T_L}{J_{eq}}$$

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Per-Phase Steady Stead Equivalent Circuit

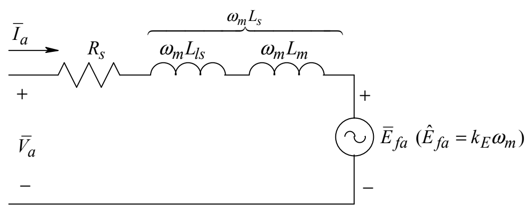

Figure 9-2 Per-phase equivalent circuit in steady state (ω_m in elect. rad/s)

$$v_{sd} = R_s i_{sd} - \omega_m L_s i_{sq}$$

$$v_{sq} = R_s i_{sq} + \omega_m L_s i_{sd} + \omega_m \lambda_{fd}$$

$$\vec{v}_s = R_s \vec{i}_s + j\omega_m L_s \vec{i}_s + j \underbrace{\sqrt{3/2}\,\omega_m \lambda_{fd}}_{\vec{e}_{fs}}$$

$$\bar{V}_a = R_s \bar{I}_a + j\omega_m L_s \bar{I}_a + j \underbrace{\omega_m \sqrt{\frac{2}{3}} \lambda_{fd}}_{\bar{E}_{fa}}$$

$$\hat{E}_{fa} = \underbrace{\sqrt{\frac{2}{3}} \lambda_{fd}}_{k_E} \omega_m = k_E \omega_m$$

$$k_E = \sqrt{\frac{2}{3}} \lambda_{fd}$$

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Controller in the dq Reference Frame

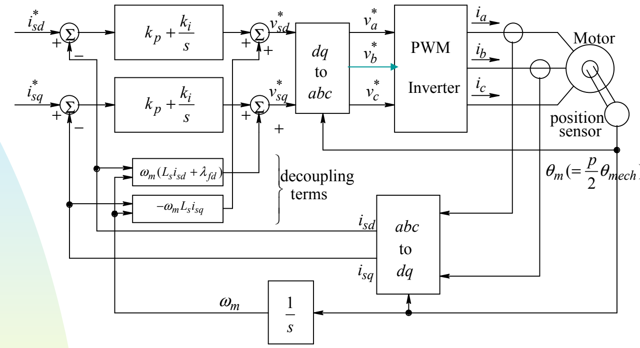

Figure 9-3 Controller in the *dq* reference frame.

$$v_{sd} = R_s i_{sd} + L_s \frac{d}{dt} i_{sd} + \underbrace{(-\omega_m L_s i_{sq})}_{\text{comp}_d} \qquad v_{sq} = R_s i_{sq} + L_s \frac{d}{dt} i_{sq} + \underbrace{\omega_m (L_s i_{sd} + \lambda_{fd})}_{\text{comp}_q}$$

$$\sqrt{|i_{sd}|^2 + |i_{sq}|^2} \le \hat{I}_{dq,rated} \left(= \sqrt{\frac{3}{2}} \hat{I}_{a,rated}\right)$$

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Vector Control of a Permanent-Magnet Synchronous-Motor Drive

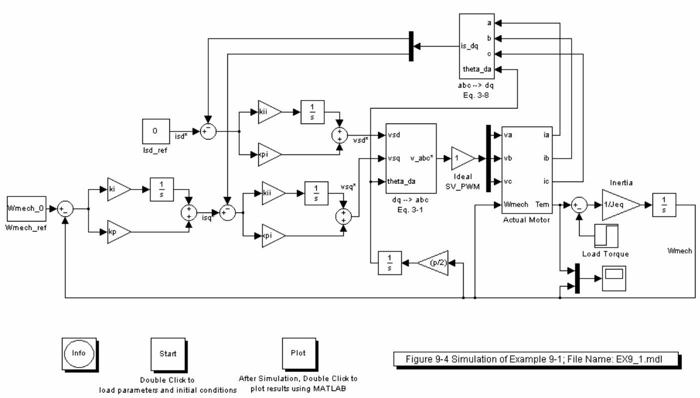

Figure 9-4 Simulation of Example 9-1; File Name: EX9_1.mdl

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Simulation Results

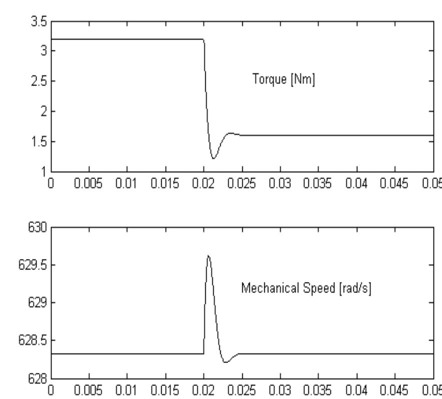

Figure 9-5 Simulation results of Example 9-1.

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Salient-Pole Synchronous Machine

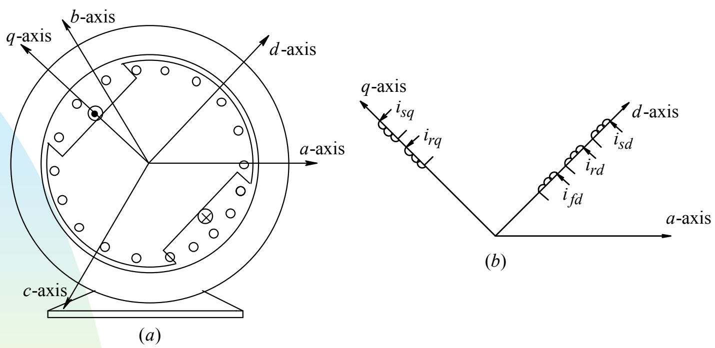

Figure 9-6 Salient-pole machine.

$$L_{sd} = L_{md} + L_{\ell s}$$
$$L_{sq} = L_{mq} + L_{\ell s}$$
$$L_{fd} = L_{md} + L_{\ell fd}$$
$$L_{rd} = L_{md} + L_{\ell rd}$$
$$L_{rq} = L_{mq} + L_{\ell rq}$$

$$\lambda_{sd} = L_{sd} i_{sd} + L_{md} i_{rd} + L_{md} i_{fd} \qquad \lambda_{rd} = L_{rd} i_{rd} + L_{md} i_{sd} + L_{md} i_{fd}$$
$$\lambda_{sq} = L_{sq} i_{sq} + L_{mq} i_{rq} \qquad \lambda_{rq} = L_{rq} i_{sq} + L_{mq} i_{sq}$$
$$\lambda_{fd} = L_{fd} i_d + L_{md} i_{sd} + L_{md} i_{rd}$$

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Winding Voltages

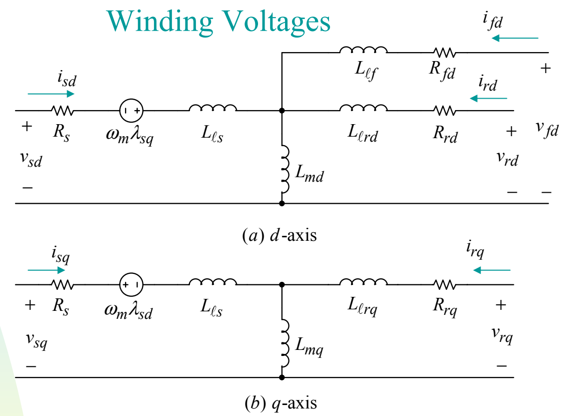

Figure 9-7 Equivalent circuits for a salient-pole machine.

$$v_{sd} = R_s i_{sd} + \frac{d}{dt}\lambda_{sd} - \omega_m \lambda_{sq}$$
$$v_{sq} = R_s i_{sq} + \frac{d}{dt}\lambda_{sq} + \omega_m \lambda_{sd}$$
$$v_{fd} = R_{fd} i_{fd} + \frac{d}{dt}\lambda_{fd}$$

$$\underbrace{v_{rd}}_{(=0)} = R_{rd} i_{rd} + \frac{d}{dt}\lambda_{rd}$$
$$\underbrace{v_{rq}}_{(=0)} = R_{rq} i_{rq} + \frac{d}{dt}\lambda_{rq}$$

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Electromagnetic Torque

$$\lambda_{sd} = L_{sd} i_{sd} + L_{md} i_{rd} + L_{md} i_{fd}$$
$$\lambda_{sq} = L_{sq} i_{sq} + L_{mq} i_{rq}$$
$$T_{em} = \frac{p}{2}(\lambda_{sd} i_{sq} - \lambda_{sq} i_{sd})$$
$$T_{em} = \frac{p}{2}[\underbrace{L_{md}(i_{fd} + i_{rd}) i_{sq}}_{\text{field+damper in d-axis}} + \underbrace{(L_{sd} - L_{sq}) i_{sd} i_{sq}}_{\text{saliency}} - L_{mq} i_{rq} i_{sd}]$$

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Space Vector Diagram in Steady State

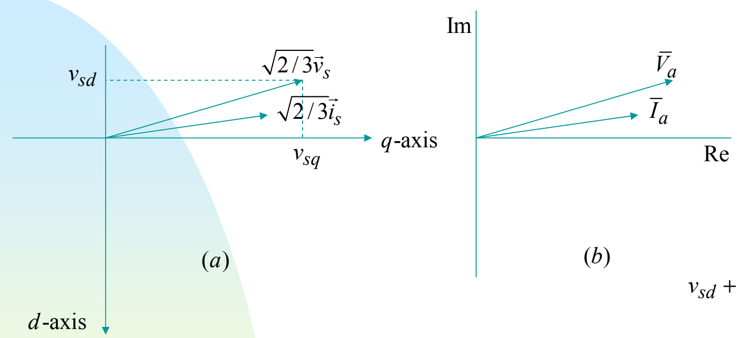

Figure 9-8 Space vector and phasor diagrams.

$$v_{sd} + j v_{sq} = \sqrt{\frac{2}{3}}\vec{v}_s$$
$$i_{sd} + j i_{sq} = \sqrt{\frac{2}{3}}\vec{i}_s$$

$$\lambda_{sd} = L_{sd} i_{sd} + L_{md} i_{fd} \qquad \lambda_{sq} = L_{sq} i_{sq}$$
$$v_{sd} = R_s i_{sd} - \omega_m L_{sq} i_{sq} \qquad v_{sq} = R_s i_{sq} + \omega_m L_{sd} i_{sd} + \omega_m L_{md} i_{fd}$$
$$v_{sd} + j v_{sq} = R_s i_{sd} + j R_s i_{sq} + j\omega_m L_{sd} i_{sd} + j\omega_m L_{md} i_{fd} - \omega_m L_{sq} i_{sq}$$

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Chapter 10

Switched-Reluctance Motor (SRM) Drives

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Cross-Section of a Switched-Reluctance Machine

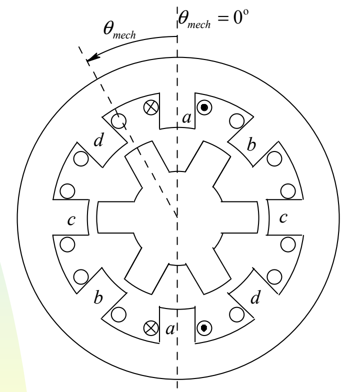

Figure 10-1 Cross-section of a four-phase 8/6 switched reluctance machine.

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Aligned and Unaligned Positions for Phase-a

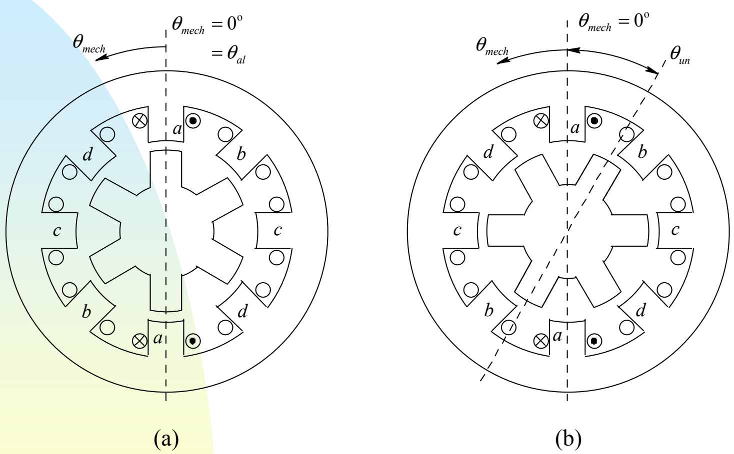

Figure 10-2 Aligned position for phase *a*; (b) Unaligned position for phase *a*.

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Typical Flux-Linkage Characteristics

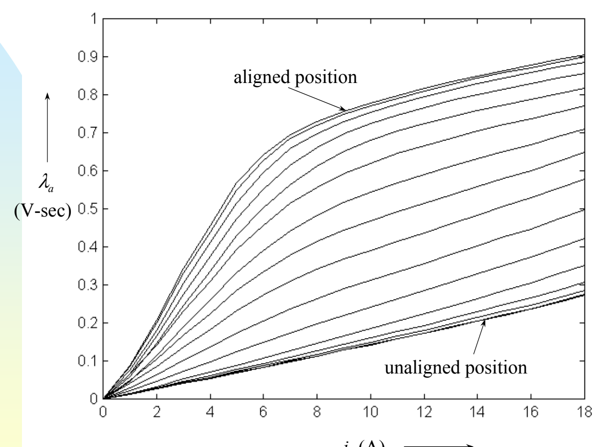

Figure 10-3 Typical flux-linkage characteristics of an SRM.

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Calculation of Torque

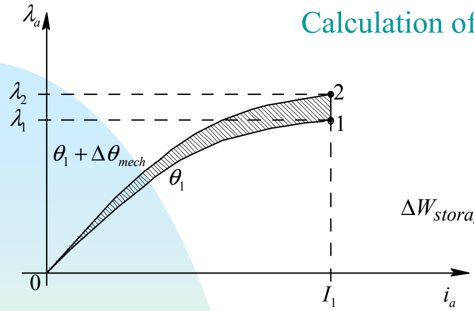

Figure 10-4 Calculation of torque.

$$\Delta W_{mech} = T_{em}\,\Delta\theta_{mech}$$

$$\Delta W_{elec} = area\,(1-\lambda_1-\lambda_2-2-1)$$

$$\Delta W_{storage} = area\,(0-2-\lambda_2-0) - area\,(0-1-\lambda_1-0)$$

$$\Delta W_{mech} = \Delta W_{elec} - \Delta W_{storage}$$

$$T_{em}\,\Delta\theta = area\,(1-\lambda_1-\lambda_2-2-1) - \{area\,(0-2-\lambda_2-0) - area\,(0-1-\lambda_1-0)\}$$

$$= \underbrace{\{area\,(1-\lambda_1-\lambda_2-2-1) + area\,(0-1-\lambda_1-0)\}}_{area\,(0-1-2-\lambda_2-0)} - area\,(0-2-\lambda_2-0)$$

$$= area\,(0-1-2-0)$$

$$T_{em} = \frac{area\,(0-1-2-0)}{\Delta\theta_{mech}} \qquad T_{em} = \left.\frac{\partial W'}{\partial\theta_{mech}}\right|_{i_a=\text{constant}}$$

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Waveforms Assuming Ideal Current Waveforms

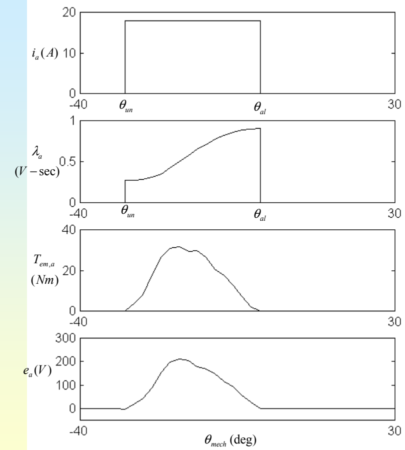

Figure 10-5 Performance assuming idealized current waveform.

$$v_a = Ri_a + e_a$$

$$e_a = \frac{d}{dt}\lambda_a(i_a, \theta_{mech})$$

$$e_a = \left.\frac{\partial\lambda_a}{\partial i_a}\right|_{\theta_{mech}} \frac{d}{dt} i_a + \left.\frac{\partial\lambda_a}{\partial\theta_{mech}}\right|_{i_a} \frac{d}{dt}\theta_{mech}$$

$$e_a = \left.\frac{\partial\lambda_a}{\partial\theta_{mech}}\right|_{i_a} \underbrace{\frac{d}{dt}\theta_{mech}}_{\omega_{mech}} = \left.\frac{\partial\lambda_a}{\partial\theta_{mech}}\right|_{i_a} \omega_{mech}$$

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Performance with a Power Processing Unit

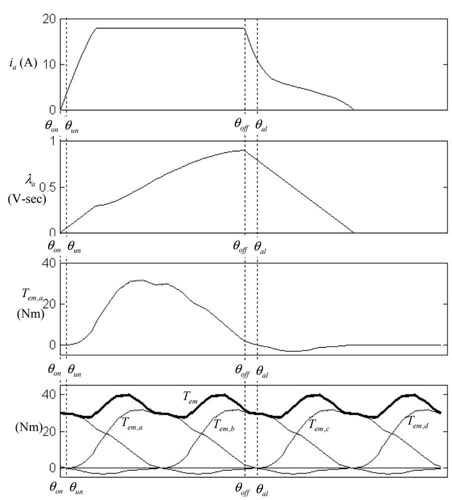

Figure 10-6 Performance with a power-processing unit.

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Role of Magnetic Saturation

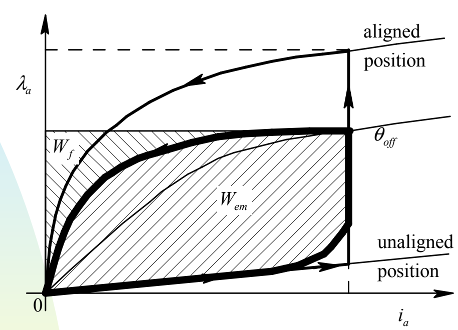

Figure 10-7 Flux-linkage trajectory during motoring.

$$Energy\,Conversion\,Factor = \frac{W_{em}}{W_{em} + W_f}$$

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Power Processing Unit

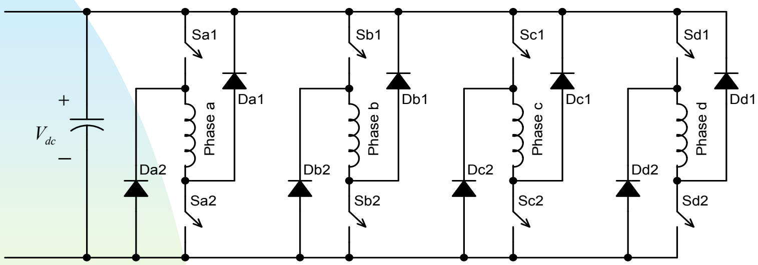

Figure 10-8 Power converter for a four-phase switched reluctance drive.

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Determining Rotor Position for Encoder-less Operation

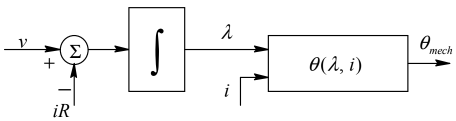

Figure 10-9 Estimation of rotor position.

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Control in Motoring Mode

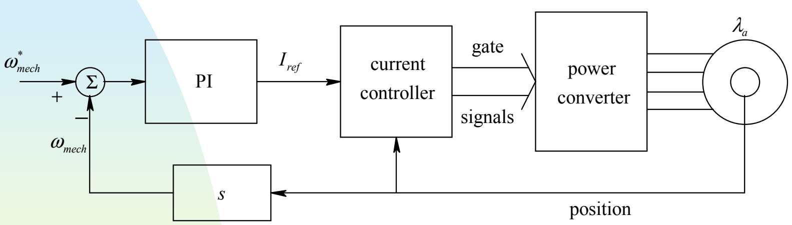

Figure 10-10 Control block diagram for motoring.

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